Turbo machine [10ME56]

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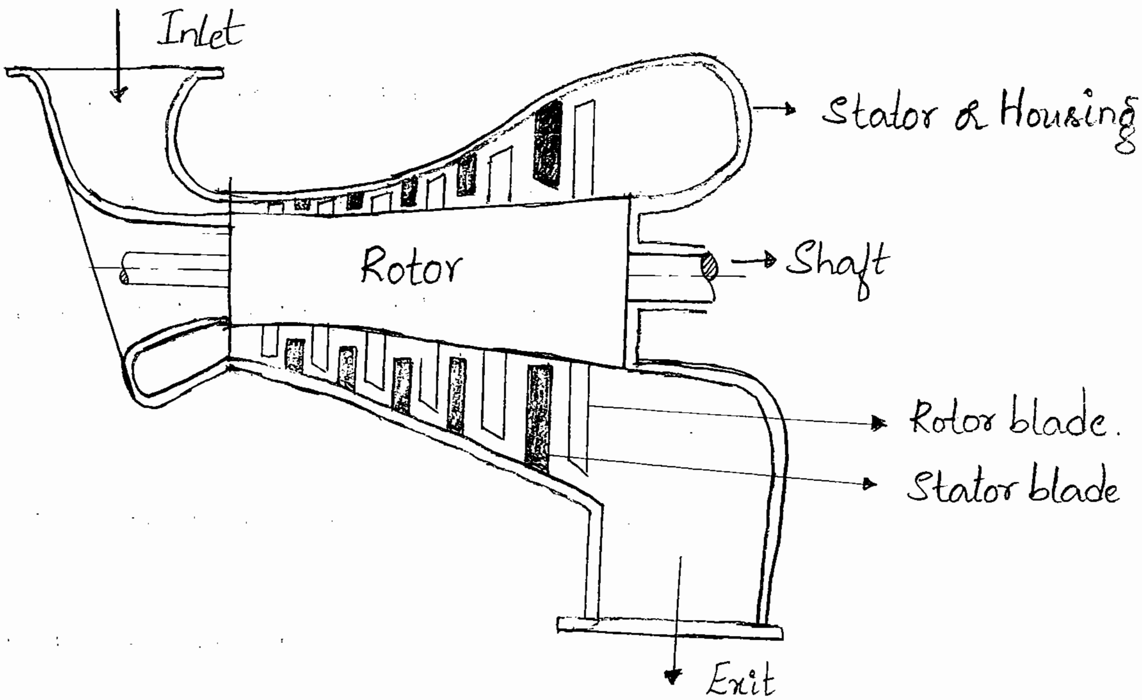
1. TURBO-MACHINE INTRODUCTION

The turbomachine is used in several applications, the primary ones being electrical power generation, aircraft propulsion and vehicular propulsion for civilian and millitary use.

Turbo is a latin word which means whirls around. Turbomachine is defined as " A turbomachine is a device in which energy transfer takes place between a flowing fluid and a rotating element due to the dynamic action and results in the change of pressure and momentum of the fluid".

Generally in turbomachines mechanical energy is transferred into or out of the system in a steady flow process. Turbomachines includes two types of machines, which are producing pressure or head, such as Centrifugal pumps, compressors, Blowers etc.. and those types which are producing power such as turbines of all kinds.

Principal components of Turbomachines



Schematic cross sectional view of a turbine showing parts.

The following are the principal components of turbomachine.

(i) A Rotor

(ii) A Stationary element

(iii) An input and/or output shaft.

(iv) A Housing.

Rotor: A Rotor carries vanes rotating in a steam of fluid flow. Depending upon the particular machine the rotor also called runner, impeller etc.., Energy transfer occurs only due to the exchange of momentum between the flowing fluid and the rotating elements.

Stationary element: A stationary element usually act as guide vanes for the proper control of flow direction during the energy conversion process. The stationary element is also known by different names among them guide blade or nozzle depending upon the particular type of machine and kind of flow occuring in it.

A Stationary element is not a necessary part of every turbomachine. [The common ceiling fan used in many buildings to circulate air and the table fan are examples of turbomachines with no stationary element. Such machines have only two elements of the four mentioned above, an input shaft and a rotating blade element].

An input and/or Output shaft: Either an input or an ~~input~~ output or both may be necessary depending on the application. In power absorbing machines, an input shaft is used whereas in power generating machines output shaft is used. In power transmitting turbomachines (example: Hydraulic couplings, clutch-plate gear drive] both input and output shaft are used.

Housing: The housing too is not a necessary part of a turbomachine. When present, it is used to restrict the fluid flow to a given space and prevent its escape in directions other than those required for energy transfer and utilization. The housing plays no role in the energy conversion process.

Types of Turbomachines

Turbomachines are mainly classified into 3 categories, they are (i) Power producing turbomachines.

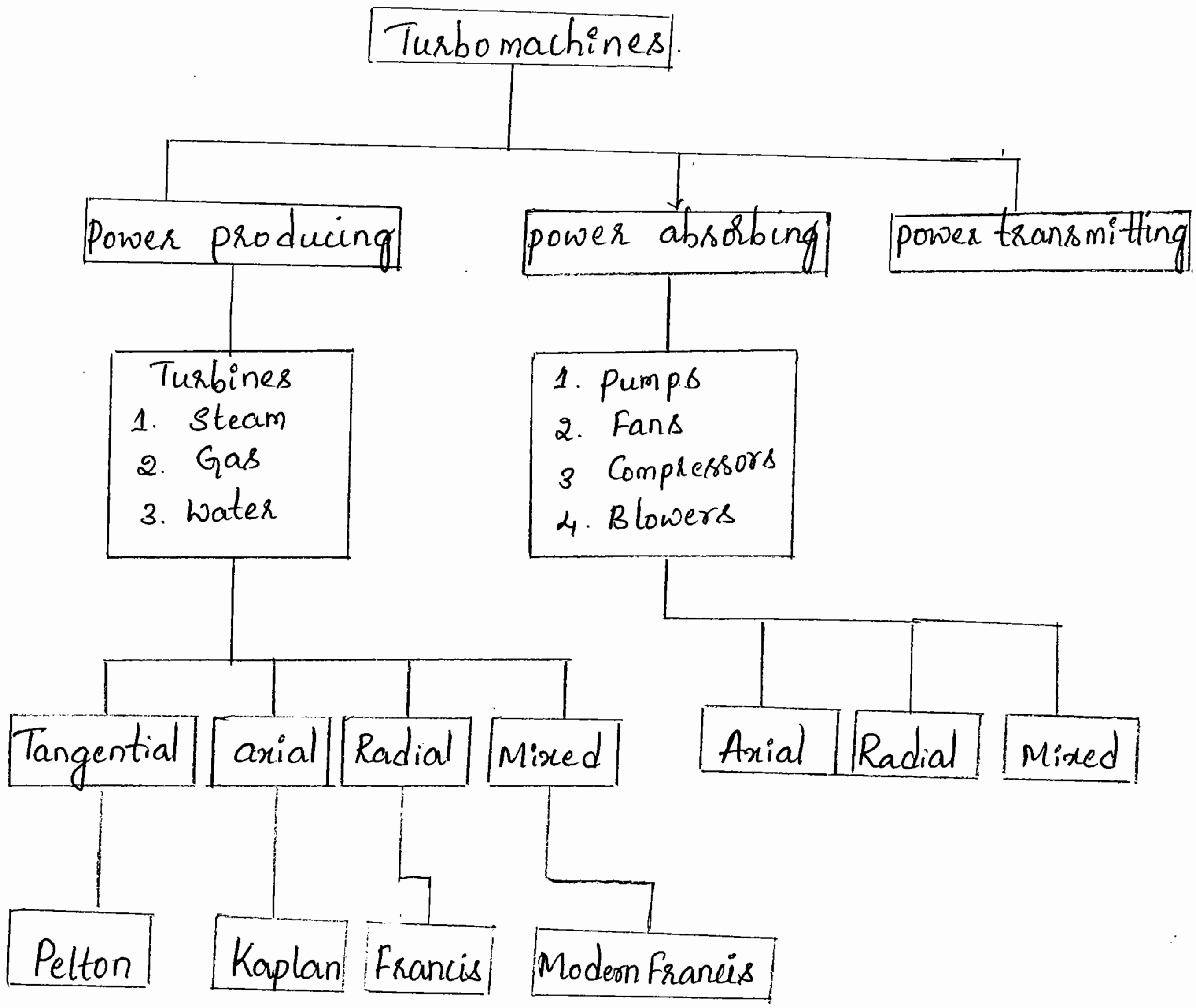
(ii) Power absorbing turbomachines.

(iii) Power transmitting turbomachines.

Power producing turbomachines: These kind of turbomachines converts fluid energy into mechanical energy.

power absorbing turbomachines: These kind of turbomachines converts mechanical energy into fluid energy.

Power transmitting turbomachines: These type of turbomachines which simply transmit power from an input shaft to an output shaft. in order to change the speed and torque on the driven member as compared with the driver.



General classification of turbomachines

In mixed flow turbomachine, the flow usually enters the rotor axially and leaves radially or vice-versa.

Positive - Displacement Machines

In a positive displacement, the interaction between the moving part and the fluid involves a change in volume and/or a translation of the fluid confined in a given boundary. During energy transfer, fluid expansion or compression may occur in a positive displacement machine without an appreciable movement of the mass centre of gravity of the confined fluid.

Comparison between positive displacement machine and Turbomachines are,

Positive displacement	Turbomachine
i) Action:	
This machine creates thermodynamics and mechanical action between a near static fluid and a relatively slow slowly moving surface and involves a volume change or displacement of the fluid.	This machine creates thermodynamics and dynamic action between a flowing fluid and a rotating element and involves energy transfer with pressure and momentum changes.

ii) <u>Operation</u>

a) It involves a reciprocating motion and unsteady flow of fluid.	a) It involves rotary motion and steady flow of fluid.
b) Since the fluid containment is positive, during stopping a certain amount of fluid trap in a state different from that of the ~~so~~ surroundings.	b) As there is no positive containment of the fluid, during stopping there is no trapping of the fluid and becomes the same state as that of the surroundings.

iii) <u>Mechanical features</u>

a) low speed machine.	a) High rotational speed.
b) Relatively complex in design.	b) Relatively simple in design.
c) It is usually heavy per unit power output.	c) Light weight per unit power output
d) Involves valve operation	d) Not employs valve operation.
e) Heavy foundation because of reciprocating masses and vibration	e) Light foundation, because of well balanced rotating

iv) Efficiency of Conversion process:

a) The use of the positive containment and anear static, energy transfer process results in a high efficiency.

a) Due to dynamic action including high-speed fluid flow, the efficiency is less compare to positive displacement machines.

b) Because of the opening and closing of valves needed for continuous operation, volumetric efficiency is low.

b) Because of no inlet and outlet valves, volumetric efficiency normally near to 100%.

c) Low fluid handling capacity

c) High fluid handling capacity.

v) Fluid phase change and Surging:

a) Problem of phase change and surging are generally of a relatively minor importance in the positive displacement machines.

a) phase change occuring during flow through a turbomachine can frequently cause serious difficulties for smooth operation, blade ~~op~~ erosion and deterioration of machine performance.

b) Surging phenomenon associated

Effect of Reynold's Number:

In a pipe flow, the Reynold's number is given by

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} \qquad \text{Since } \nu = \frac{\mu}{\rho}$$

Where V is the mean velocity of the fluid and D is the diameter pipe. It is a important parameter to characterize the nature of flow. If $Re < 2000$, then flow is laminar.

If $Re > 4000$, then flow is turbulent.

But in turbo machines, the Reynold's number, is not such an important parameter since the machine losses are not determined by the viscous effect alone. Because various other losses such as due to shock at entry, turbulence, impact and leakage, are also to be accounted along with the viscous resistance or friction.

Most of the turbomachines use relatively low viscous fluids like air, steam, water and light oils. As a result, the flow in a turbo machine is generally a turbulent in nature.

Under varying load conditions, Moody's has suggested an equation to determine turbine efficiency

$$\eta_p = \left[1 - (1-\eta_m)\left(\frac{D_m}{D_p}\right)^{1/5}\right] \qquad (a)$$

where η_p = Efficiency of the prototype of diameter D_p

η_m = Efficiency of the model of diameter, D_m

First and Second law of thermodynamics applied to Turbomachines.

Fluid flow in a turbomachine is assumed to be steady and this assumptions permits the analysis of energy and mass transfer by using the steady state energy equation, and from the First Law of thermodynamics,

$$Q+\dot{m}\left(h_1+\frac{V_1^2}{2}+gz_1\right) = P+\dot{m}\left(h_2+\frac{V_2^2}{2}+gZ_2\right) \quad -(1)$$

Here, Q = Rate of heat flow.

P = Power output

$\dot{m}$ = mass flow rate

Divide equation (1) by $\dot{m}$, also $\frac{Q}{\dot{m}} = q$, $\frac{P}{\dot{m}} = w$

where q = heat transfer per unit mass flow

w = work transfer per unit mass flow.

(1) $\Rightarrow$
$$q - w = \left(h_2+\frac{V_2^2}{2}+gz_2\right) - \left(h_1+\frac{V_1^2}{2}+gz_1\right) \quad -(2)$$

where h_1 and h_2 are static enthalpies at inlet and outlet of turbomachine.

(~~Stagnation~~ Static enthalpy: Whenever the kinetic and potential of fluid are negligible, then the energy of fluid is called static enthalpy. ie combination of internal energy and pressure and volume product, $h = u + pv$].

also $$h_1 + \frac{V_1^2}{2} + gz_1 = h_{01}$$

$$h_2 + \frac{V_2^2}{2} + gz_2 = h_{02}$$

where h_{01} & h_{02} are stagnation enthalpies at inlet and outlet of turbomachine.

[Stagnation state is defined is defined as a state in a fluid flow field, when the fluid is brought to rest isentropically. Entholpy of fluid at that state is called stagnation enthalpy].

(2) $\Rightarrow$ $$q - w = h_{02} - h_{01} = dh_0$$

Since most of turbomachines operate at room temperature there is no heat transfer takes place ie $q = 0$

$$\boxed{-w = dh_0} \text{ or } \boxed{-w = \Delta h_0}$$

In power generating machines, w is positive as defined so that Δh_0 is negative [i.e., the stagnation enthalpy at the exit of the machine is less than that at the inlet]

$h_{0_2} - h_{0_1}$ is negative.

$\Rightarrow \quad h_{0_2} < h_{0_1}$ (Enthalpy is reduced)

$\Rightarrow \quad \boxed{w = h_{0_1} - h_{0_2}}$

In power absorbing machines, w is negative, Δh_0 is positive, ie $h_{0_2} > h_{0_1}$ (enthalpy is increased).

$$\boxed{-w = h_{0_2} - h_{0_1}}$$

Considering Second law of thermodynamics, applying stagnation state to change,

$$T_0 ds_0 = dh_0 - v_0 dp_0 \quad — (3)$$

where T_0 = Temperature of fluid at stagnation state

v = Specific volume, ds_0 = Entropy change.

but we know, $dh_0 = -w$.

(3) $\Rightarrow \quad -w = T_0 ds_0 + v_0 dp_0$

The work transfer is maximum when change in entropy is minimum.

Dimensionless parameters and Significance:

Performance of a turbomachine depends on the following variables; Discharge Q, Speed or RPM (N), Size or Rotor diameter (D), Energy per unit mass flow (gH), power (P), Density of fluid (ρ) Dynamic viscosity of fluid (μ). Using the dimensional analysis obtain the π-numbers.

Performance of turbomachine depends on.

(1) Discharge — Q — m^3/s — L^3T^{-1}

(2) Speed — N — RPM (s^{-1}) — T^{-1}

(3) Diameter — D — m — L

(4) Energy per unit mass flow - gH — m^2/s^2 — L^2T^{-2}

(5) Power — P — Nm/s — ML^2T^{-3}

(6) Density of fluid — ρ — kg/m^3 — ML^{-3}

(7) Viscosity — μ — Ns/m^2 — $ML^{-1}T^{-1}$

$$f(Q, N, D, gH, P, \rho, \mu) = \text{constant}.$$

No. of variables, $n = 7$

No. of fundamental variables $m = 3$

No. of π- terms $11-111 = 4$

Select D, N, ρ are repeating variables

$$f(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

<u>π_1-term</u> . $\pi_1 = D^{a_1} N^{b_1} \rho^{c_1} Q$

$$M^0 L^0 T^0 = L^{a_1} (T^{-1})^{b_1} (ML^{-3})^{c_1} L^3 T^{-1}$$

Equating the powers of M.L.T on both sides we get.

For M, $0 = c_1$

For L, $0 = a_1 - 3c_1 + 3 \Rightarrow a_1 = -3$

for T, $0 = -b_1 - 1 \Rightarrow b_1 = -1$

$\therefore$ $\pi_1 = D^{-3} N^{-1} \rho^0 Q$.

$$\pi_1 = \frac{Q}{ND^3} \quad \text{—— (1)}$$

<u>π_2-term.</u>

$$\pi_2 = D^{a_2} N^{b_2} \rho^{c_2} gH$$

$$M^0 L^0 T^0 = L^{a_2} (T^{-1})^{b_2} (ML^{-3})^{c_2} L^2 T^{-2}$$

Equating powers of M, L, T on both sides we get

For M, $0 = c_2$

For L, $0 = a_2 - 3c_2 + 2 \Rightarrow a_2 = -2$

For T, $0 = -b_2 - 2$ & $b_2 = -2$

$$\therefore \quad \pi_2 = D^{-2} N^{-2} \rho^{0} g H$$

$$\pi_2 = \frac{gH}{N^2 D^2} \quad \text{—— (2)}$$

<u>π_3 - term</u>

$$\pi_3 = D^{a_3} N^{b_3} \rho^{c_3} P$$

$$M^0 L^0 T^0 = L^{a_3} (T^{-1})^{b_3} (ML^{-3})^{c_3} ML^2 T^{-3}$$

Equating powers of MLT on b.s.

For M, $0 = c_3 + 1$, $\Rightarrow c_3 = -1$

For L, $0 = a_3 - 3c_3 + 2 \Rightarrow a_3 = -5$

For T, $0 = -b_3 - 3 \Rightarrow b_3 = -3$

$$\pi_3 = D^{-5} N^{-3} \rho^{-1} P$$

$$\pi_3 = \frac{P}{\rho N^3 D^5} \quad \text{—— (3)}$$

<u>π_4 - term</u> $\quad \pi_4 = D^{a_4} N^{b_4} \rho^{c_4} \mu$

$$M^0 L^0 T^0 = L^{a_4} (T^{-1})^{b_4} (ML^{-3})^{c_4} ML^{-1} T^{-1}$$

Equating powers of MLT on both sides.

For M, $0 = c_4 + 1 \Rightarrow c_4 = -1$

For L, $0 = a_4 - 3c_4 - 1 \Rightarrow a_4 = -2$

For T, $0 = -b_4 - 1 \Rightarrow b_4 = -1$

$$\therefore \pi_4 = D^{-2} N^{-1} \rho^{-1} \mu$$

$$\pi_4 = \frac{\mu}{\rho N D^2} \quad \text{—— (4)}$$

Significance of π-terms

1) Flow co-efficient or Capacity coefficient or Specific capacity.

The term $\pi_1 = \frac{Q}{ND^3}$ is the capacity coefficient & flow coefficient or specific capacity which signifies the volumetric flow rate of fluid through a turbo machine of unit diameter of ~~running~~ runner operating at unit speed. The specific capacity is constant for similar rotors.

Specific capacity is given by $\pi_1 = \frac{Q}{ND^3} \propto \frac{D^2 V}{ND^3} \propto \frac{V}{ND}$

$$\pi_1 = \frac{V}{u} = \frac{1}{\phi} \quad , \text{where } \phi = \frac{u}{V}, \text{ speed ratio}$$

Head coefficient or Specific head.

The term $\pi_2 = \frac{gH}{N^2D^2} = \frac{gH}{U^2} = \frac{H}{(U^2/g)}$ [$\because$ ND = U] is called the head coefficient.

It represents the ratio of the kinetic energy of the fluid under head H to kinetic energy of fluid running at the tangential speed of the rotor. The head coefficient is constant for simillar impellers. For a machine of given impeller diameter the head varies directly as the square of the tangential speed of rotor or impeller. ie., $H \propto V^2$

Power co-efficient or Specific power

The term $\pi_3 = \frac{P}{\rho N^3 D^5}$ is called the power co-efficient or Specific power. It represents, the relation between the power, fluid density, speed and wheel diameter. For a given machine, the power is directly proportional to the cube of the speed of runner or rotor.

$$\pi_3 = \frac{P}{\rho N^3 D^5}$$

ie $\quad P \propto N^3$

Specific Speed (Ω)

The specific speed is only the parameter that does not contain the linear dimension of the runner. Hence while operating under the same conditions of flow and head, all geometrically simillar machines have the same specific speed, irrespective of their sizes.

The specific speed can be expressed in terms of discharge for power absorbing machine and the power for power generating machine.

Specific speed of a pump

The equation of specific speed for a pump can be obtained by manipulating flow coefficient and head coefficient.

ie $$\pi_5 = \frac{(\pi_1)^{1/2}}{(\pi_2)^{3/4}} = \left(\frac{Q}{ND^3}\right)^{1/2} \times \left(\frac{D^2N^2}{gH}\right)^{3/4} = \Omega$$

$$\Omega = \frac{Q^{1/2}}{N^{1/2}D^{3/2}} \cdot \frac{D^{3/2}N^{3/2}}{g^{3/4}H^{3/4}}$$

$$\Omega = \frac{NQ^{1/2}}{(gH)^{3/4}}$$

This can also be expressed as

$$\boxed{N_s = \frac{NQ^{1/2}}{H^{3/4}}} \text{ ...... rad/s}$$

Specific speed of pump is defined as "a speed of geometrically similar machines discharging $1 m^3/s$ of water under a head of one meter."

<u>Alternative method</u>

Head co-efficient is given by $\frac{gH}{N^2D^2}$

$\therefore \quad H \propto N^2 D^2$

or $\quad D^2 \propto \frac{H}{N^2}$

$$D \propto \frac{H^{1/2}}{N} \quad \text{—— (a)}$$

Also, the flow co-efficient is $\frac{Q}{ND^3}$

$Q \propto ND^3$

$$Q \propto \frac{N(H^{1/2})^3}{(N)^3} \propto \frac{N H^{3/2}}{N^3}$$

$$Q \propto \frac{H^{3/2}}{N^2}, \qquad Q = C\frac{H^{3/2}}{N^2}$$

Where C is a proportionality constant whose value is to be found out from the definition of specific speed of pump

From definition, $N_s = N$ at $Q = 1m^3/s$, $H = 1m$

ie from above equation

$$1 = C \frac{1^{3/2}}{N_s^2}$$

$$\Rightarrow \boxed{C = N_s^2}$$

ie $$Q = \frac{N_s^2 H^{3/2}}{N^2}$$

$$N_s^2 = \frac{Q N^2}{H^{3/2}}$$

$$\Rightarrow \boxed{N_s = \frac{Q^{1/2} N}{H^{3/4}}}$$

Specific speed of a turbine

A specific speed for a turbine can be obtained with the help of head coefficient and power coefficient terms,

$$\pi_6 = \frac{\pi_1^{1/2}}{\pi_2^{5/4}} = \left(\frac{P}{\rho N^3 D^5}\right)^{1/2} \times \left(\frac{N^2 D^2}{gH}\right)^{5/4}$$

$$= \frac{N P^{1/2}}{\rho^{1/2} g^{5/4} H^{5/4}},$$ where $\rho^{1/2} g^{5/4}$ is constant,

$$\Rightarrow \boxed{N_s = \frac{N P^{1/2}}{H^{5/4}}}$$

Specific speed of a turbine is defined as a "Speed of a geometrically simillar machines which produce 1kw power under a head of 1m".

<u>Alternative method</u>.

power coefficient is given by $\frac{P}{\rho N^3 D^5}$

$$P \propto \rho N^3 D^5 \text{ or } P \propto N^3 D^5 \quad \text{— (1)}$$

also $$\frac{gH}{N^2 D^2}, \; H \propto N^2 D^2 \quad \text{— (A)}$$

$$D^2 \propto H / N^2$$

$$D \propto \frac{H^{1/2}}{N} \quad \text{— (a)}$$

Substitute (a) in (1).

$$P \propto N^3 \left(\frac{H^{1/2}}{N}\right)^5 \propto \frac{H^{5/2}}{N^2}$$

$$P = C \frac{H^{5/2}}{N^2}, \quad \text{where } C \text{ is proportionality constant}$$

ie at $P = 1\,kw$, $H = 1m$ $N = N_s$.

$$\Rightarrow \quad C = N_s^2, \Rightarrow \quad N_s = \frac{N P^{1/2}}{H^{5/4}}$$

Importance of specific speed

It is very important parameter to select a particular type of machines and for its designing aspects, because of any machine's posses maximum efficiency means it has high specific speed value.

Range of specific speeds of different turbomachines.

Turbomachines	Specific speed 'Ns'.
1. Pelton wheel (Impulse)	
a. Single jet	3 – 30
b. Double jet	14 – 43
c. Four jet	20 – 60
2. Francis turbine (Reaction)	
a. Radial flow (Slow speed)	60 – 102
b. Mixed flow (medium speed)	102 – 188
c. Mixed flow (high speed)	188 – 368
3. Propeller turbine	256 – 518
4 Kaplan turbine	428 – 856
5. Axial flow steam and gas turbine	18 – 160

6.	Centrifugal pump	
	a. Slow speed	12 - 25
	b. Medium speed	20 - 50
	c. High speed	50 - 95
7.	Mixed flow pump	95 - 210
8.	Axial flow pump	170 - 320
9.	Radial flow compressors	21 - 74
10.	Axial flow compressors, Blowers..,	74 - 1050

Unit Quantities

In hydraulic turbines, it is usual to define quantities referred to as unit flow, unit power, unit speed etc..., which are the values of the quantities under consideration per unit head. They are usually used in turbine design.

Unit flow

Unit flow is the flow that occurs through the turbine while working under unit head, assuming operation at design speed and efficiency.

we have, from continuity equation, $Q = AV$.

but A is constant $\Rightarrow Q \propto V$ —— (i)

again we know, $V = \dfrac{\pi D N}{60}$, Since D = constant

$$V \propto N \quad \text{—— (iii)}$$

also $V = C_v\sqrt{2gH}$

ie $V \propto \sqrt{H}$ —— (ii)

from (i) and (ii) $Q \propto \sqrt{H}$

$$Q = K\sqrt{H}$$

From definition, $Q = Q_u$ at $H = 1m$.

ie $Q_u = K\sqrt{1}$, $K = Q_u$.

$$\Rightarrow \quad Q = Q_u\sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}} = \text{constant}.$$

unit Speed

Unit speed is that speed at which the machines runs under unit head.

we have from (ii) and (iii)

$$N \propto \sqrt{H}$$

$$N = K\sqrt{H}$$

From definition, $N = N_u$ at $H = 1m$

ie $N_u = K\sqrt{1}$

$K = N_u$

ie $\quad N = N_u\sqrt{H}$

$$\boxed{N_u = \frac{N}{\sqrt{H}}} = \text{constant}$$

Unit power

Unit power is the power developed by the hydraulic (turbine) machine while working under a unit ~~speed~~ head, assuming operation at design speed and efficiency.

we have $\quad P = \eta \dfrac{\rho g Q H}{1000} = \ldots \text{kW}$

$\Rightarrow \quad P \propto QH$

from i) & ii) $\quad Q \propto \sqrt{H}$

$\Rightarrow \quad P \propto \sqrt{H}\,H, \quad P \propto H^{3/2}$

or $\quad P = K H^{3/2}$

From definition, $P = P_u$ at $H = 1m$

$P_u = K\, 1^{3/2}, \quad \boxed{K = P_u}$

$\Rightarrow \quad P = P_u H^{3/2}$

$$\boxed{P_u = \frac{P}{H^{3/2}}}$$

1. A turbine develops 9000 kW working at a head of 30m and running at 100 rpm. If the head is reduced by 12m. Determine the speed and power developed by turbine.

Given, $P_1 = 9000 kW$

$H_1 = 30m$

$N_1 = 100 rpm$

$H_2 = 18m$, $N_2 = ?$ $P_2 = ?$

we have unit speed $N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$

$$\frac{100}{\sqrt{30}} = \frac{N_2}{\sqrt{18}}$$

$N_2 = 77.46 rpm$.

again, unit power

$$P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}.$$

$$\frac{9000}{30^{3/2}} = \frac{P_2}{18^{3/2}}$$

$P_2 = 4182.82 kW$

2. An axial-flow pump with a rotor diameter of 300mm handles liquid water at the rate of $160 m^3/h$ while operating at 1500RPM. The corresponding head ~~energy~~ input is $125 J/kg (gH)$. If a second geometrically simillar pump with a diameter of 200mm operates at 3000RPM. what are its (a) flow rate (b) power input ~~and output~~ for first and second pumps.

Given: Axial flow pump.

First pump. $D_1 = 300mm$, $Q_1 = 160 m^3/h$, $N_1 = 1500RPM$

$gH_1 = 125 J/kg$

Second pump $D_2 = 200mm$, $N_2 = 3000RPM$

To find, $P_1 = ?$, Q_2, P_2

wkT.

$$P_1 = \frac{\rho g Q H_1}{1000} = \ldots kW$$

Q in m^3/s.

$$= \frac{1000 \times 160 \times 125}{3600 \times 1000}$$

$P_1 = \underline{\underline{5.56 kW}}$ input power for first pump.

Since for simillar (rotors) machines, flow co-efficient is constant ie.

$$\pi_1 = \frac{Q}{ND^3} = constant$$

ie $$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}$$

$$\frac{160}{1500 \times 300^3} = \frac{Q_2}{3000 \times 200^3}$$

$$Q_2 = 94.815 \ m^3/h.$$

also Head coefficient $\pi_2 = \dfrac{gH}{N_1^2 D^2} = \text{constant}$

$$\frac{(gH)_1}{N_1^2 D_1^2} = \frac{(gH)_2}{N_2^2 D_2^2}.$$

$$\frac{125}{1500^2 \times 300^2} = \frac{(gH)_2}{3000^2 \times 200^2}$$

$$(gH)_2 = 222.22 \ J/kg$$

For geometrically ~~s~~imillar machines power co-efficient is constant.

$$\pi_3 = \frac{P}{\rho D^5 N^3} = \text{constant}$$

$$\frac{P_1}{\rho N_1^3 D^5} = \frac{P_2}{\rho N_2^3 D^5}$$

$$\frac{5.56}{1500^3 \times 300^5} = \frac{P_2}{3000^3 \times 200^5}$$

$P_2 = \underline{\underline{5.85}}$

Alternatively:

$$P_2 = \rho Q_2 g H_2$$

$$= 1000 \times \frac{94.815}{3600} \times 222.22$$

$$P_2 = \underline{\underline{5.853 \text{ kW}}}$$

3. An output of 10 kW was recorded on a turbine, 0.5 m diameter revolving at a speed of 800 rpm, under a head of 20 m. What is the diameter and output of another turbine which works under a head of 180 m at a speed of 200 rpm when their efficiencies are same. Find the specific speed and name the turbine can be used.

First turbine, $P_1 = 10\text{kw}$, $D_1 = 0.5\text{m}$, $N_1 = 800\text{rpm}$, $H_1 = 20\text{m}$

Second turbine, $H_2 = 180\text{m}$, $N_2 = 200\text{rpm}$, $P_2 = ?$, $D_2 = ?$

Efficiency is same means Specific speed is same.

For simillar impellers, head, flow and power co-efficient are same. ie

$$\frac{gH_1}{N_1^2 D_1^2} = \frac{gH_2}{N_2^2 D_2^2}$$

$$\frac{20}{800^2 \times 0.5^2} = \frac{180}{200^2 \times D_2^2}$$

$$\underline{\underline{D_2 = 6m}}$$

also

$$\frac{P_1}{\rho N_1^3 D_1^5} = \frac{P_2}{\rho N_2^3 D_2^5}$$

$$\frac{10}{800^3 \times 0.5^5} = \frac{P_2}{200^3 \times 6^5}$$

$$P_2 = \underline{\underline{38880}}\,kW$$

we have Specific Speed.

$$N_S = \frac{N_1\sqrt{P_1}}{H_1^{5/4}} = \frac{N_2\sqrt{P_2}}{H_2^{5/4}}$$

$$= \frac{800\sqrt{10}}{20^{5/4}} = 59.8139$$

$$N_S = \underline{\underline{59.814}}$$

4. A pelton wheel is running at a speed of 200RPM and develops 5200kW when working under a head of 220m with an

unit power and ~~uni~~ specific speed. Find the speed, flow and power when its operating point changes to a head of 140m. Take density of water = 1000kg/m³.

Given,

$N_1 = 200\,rpm, \quad P_1 = 5200\,kw, \quad H_1 = 220m, \quad \eta = 0.8$

$H_2 = 140m.$

we have unit speed $N_u = \dfrac{N_1}{\sqrt{H_1}}$

$$N_u = \frac{200}{\sqrt{220}} = \underline{\underline{13.48\,RPM}}.$$

we know, $P_1 = \eta \dfrac{\rho g Q_1 H_1}{1000} = \ldots kw.$

$$5200 = \frac{0.8 \times 9.81 \times 1000 \times Q_1 \times 220}{1000}$$

$$Q_1 = 3.01\ m^3/s$$

we have unit flow $Q_u = Q_1/\sqrt{H_1}$

$$= \frac{3.01}{\sqrt{220}}$$

$$Q_u = \underline{\underline{0.203\ m^3/s}}$$

also $P_u = \dfrac{P_1}{H_1^{3/2}} = \dfrac{5200}{220^{3/2}}$

$$P_u = \underline{\underline{1.594\,kW}}$$

Specific Speed. $N_s = \frac{N_1\sqrt{P_1}}{(H_1)^{5/4}} = \frac{200\sqrt{5200}}{(220)^{5/4}}$

$$\underline{\underline{N_s = 17.02}}$$

Speed, flow and power at $H = 140m$.

unit speed is same for a machine operating at another operating point.

$$\therefore \; N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$N_2 = \frac{200\sqrt{140}}{\sqrt{220}}$$

$$N_2 = \underline{\underline{159.55}}\,RPM$$

also $$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$Q_2 = \frac{3.01\sqrt{140}}{\sqrt{220}}$$

$$Q_2 = \underline{\underline{2.401\,m^3/s}}$$

again $$P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$P_2 = \underline{\underline{2639.74\,kW}}$$

we know, Specific Speed, $N_s = \frac{N_2\sqrt{P_2}}{H_2^{5/4}} = \underline{\underline{17.02}}$

1. The thrust (T) of a propeller is assumed to depend on the axial velocity of the fluid V, the density ρ and viscosity μ of fluid, the speed N in RPM and the diameter D. Find the relationship of T by dimensional analysis.

Thrust is depends on

Velocity V. — LT^{-1}

Density ρ, — ML^{-3}

Viscosity μ — $ML^{-1}T^{-1}$

Speed N — T^{-1}

Diameter D — L

and thrust T — MLT^{-2} (N)

The functional form is

$$f(T, V, D, \mu, \rho, N) = \text{constant}$$

Here no. of variables, $n = 6$

no. of fundamental variables $m = 3$

no. of π-terms required $n - m = 3$

Assume D, N, ρ are repeating variables.

ie $f(\pi_1, \pi_2, \pi_3) = \text{constant}$

$\pi_1 = D\ N\ \rho\ T$

Substituting dimensions on both sides.

$$M^0L^0T^0 = L^{a_1}(T^{-1})^{b_1}(ML^{-3})^{c_1}\ MLT^{-2}$$

Equating the powers of MLT, we get

For M, $\quad 0 = c_1 + 1 \quad \Rightarrow \quad c_1 = -1$

For L, $\quad 0 = a_1 - 3c_1 + 1$

For T, $\quad 0 = -b_1 - 2 \quad \Rightarrow \quad b_1 = -2$

$$0 = a_1 - 3(-1) + 1$$

$$0 = a_1 + 4, \quad a_1 = -4$$

$$\Rightarrow \quad \pi_1 = D^{-4}N^{-2}\rho^{-1}T = \frac{T}{\rho N^2 D^4} \quad \text{--- (1)}$$

$$\pi_2 = D^{a_2}N^{b_2}\rho^{c_2}V$$

$$M^0L^0T^0 = L^{a_2}(T^{-1})^{b_2}(ML^{-3})^{c_2}\ LT^{-1}$$

Equating the powers of MLT, we get

For M, $\quad 0 = c_2$

For L, $\quad 0 = a_2 - 3c_2 + 1, \Rightarrow a_2 = -1$

For T, $\quad 0 = -b_2 - 1, \Rightarrow b_2 = -1$

$$\therefore \quad \pi_2 = D^{-1}N^{-1}\rho^{0}V$$

$$\pi_2 = \frac{V}{ND} \quad \text{--- (2)}$$

$$\pi_3 = D\ N\ \rho\ \mu$$

$$M^0L^0T^0 = L^{a_3}(T^{-1})^{b_3}(ML^{-3})^{c_3}\ ML^{-1}T^{-1}$$

Equating the powers of MLT, we get

For M, $0 = c_3 + 1$, $c_3 = -1$.

For L, $0 = a_3 - 3c_3 - 1$, $a_3 = -2$

For T, $0 = -b_3 - 1$, $b_3 = -1$

$$\therefore \quad \pi_3 = D^{-2} N^{-1} \rho^{-1} \mu$$

$$\pi_3 = \frac{\mu}{\rho N D^2}$$

Now, the functional relationship is

$$\pi_1 = f(\pi_2 , \pi_3)$$

$$\frac{T}{\rho N^2 D^4} = f\left(\frac{V}{ND} , \frac{\mu}{\rho N D^2}\right)$$

$$T = \rho N^2 D^4 \ f\left(\frac{V}{ND} , \frac{\mu}{\rho N D^2}\right)$$

2. The pressure drop (Δp) in a pipe depends upon the mean velocity flow (V), length of pipe L, dia of pipe D, viscosity μ, Density of fluid ρ, average height of roughness of projection on side of pipe surface K. By using dimensional analysis, obtain an expression ~~on side~~ for Δp. Show that head loss $h_f = \dfrac{4fLV^2}{2gD}$

The functional relationship is

$$\Delta p = f[L, V, D, \mu, \rho, K]$$

General relationship is

$$f(\Delta p, L, V, D, \mu, \rho, K) = 0$$

No. of variables, $n = 7$

No. of primary variables, $m = 3$

No. of π-terms Required, $(n-m) = 4$

Repeating variables are. D, V, ρ

Dimensions:

Δp - Pa — N/m^2 —— $ML^{-1}T^{-2}$

L — m —— L

V — m/s —— LT^{-1}

D — m —— L

μ — $N.s/m^2$ —— $ML^{-1}T^{-1}$

ρ —— kg/m^3 —— ML^{-3}

K — m —— L

Let $\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} \Delta p$

$$M^0L^0T^0 = L^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} ML^{-1}T^{-2}$$

Equating powers of MLT on both sides, we get.

For M, $0 = c_1 + 1$, $c_1 = -1$

For L, $0 = a_1 + b_1 - 3c_1 - 1$, $a_1 = 0$

For T, $0 = -b_1 - 2$. $b_1 = -2$

$$\pi_1 = D^{\circ} V \rho \Delta P$$

$$\pi_1 = \frac{\Delta P}{\rho V^2}$$

Let π_2- term, $\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} L$

$$M^0 L^0 T^0 = L^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} L$$

Equating powers of MLT, we get

For M, $0 = c_2$, $c_2 = 0$

For L, $0 = a_2 + b_2 - 3c_2 + 1$, $\Rightarrow a_2 = -1$

For T, $0 = -b_2$. $b_2 = 0$

$$\pi_2 = D^{-1} V^0 \rho^0 L$$

$$\pi_2 = \frac{L}{D}$$

Let $\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} \mu$

$$M^0 L^0 T^0 = L^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} ML^{-1} T^{-1}$$

Equating powers of MLT, we get

For M, $0 = c_3 + 1$, $c_3 = -1$

For L $0 = a_3 + b_3 - 3c_3 - 1$

For T, $0 = -b_3 - 1$, $b_3 = -1$

$0 = a_3 - 1 - 3(-1) - 1$, $a_3 = -1$

$$\pi_3 = D^{-1} V^{-1} \rho^{-1} \mu$$

$$\pi_3 = \frac{\mu}{\rho V D}$$

π_4

$$M^0L^0T^0 = L^{a_4}(LT^{-1})^{b_4}(ML^{-3})^{c_4} L$$

Equating powers of MLT, we get

For M, $0 = c_4$

For L, $0 = a_4 + b_4 - 3c_4 + 1$

For T, $0 = -b_4$, $b_4 = 0$

$$\Rightarrow a_4 = -1$$

$$\pi_4 = D^{-1} V^0 \rho^0 K$$

$$\pi_4 = \frac{K}{D}$$

$\therefore$ Functional relationship is given by

$$\pi_1 = f(\pi_2, \pi_3, \pi_4)$$

$$\frac{\Delta P}{\rho V^2} = f\left[\frac{L}{D}, \frac{\mu}{\rho V D}, \frac{K}{D}\right]$$

Since pressure drop mainly depends on length of the pipe, so we can take the term $\frac{L}{D}$ out of the function.

ie $$\frac{\Delta P}{\rho V^2} = \frac{L}{D} f\left[\frac{\mu}{\rho V D}, \frac{K}{D}\right]$$

$$\Delta p = \frac{\rho L V^2}{D} f\left[\frac{\mu}{\rho V D}, \frac{K}{D}\right]$$

but we know $\frac{\Delta P}{\rho g} = h_f$.

$$\Rightarrow \Delta p = h_f \cdot \rho g.$$

$$\Rightarrow \quad h_f \cdot \rho g = \frac{\rho L V^2}{D} f\left(\frac{\mu}{\rho V D}, \frac{K}{D}\right)$$

and relative roughness ($\epsilon = K/D$).

$$\therefore \quad f = f(R, \epsilon)$$

$$\text{let } f(R.\epsilon) = 4f/2$$

$$\therefore \quad h_f = \frac{LV^2}{gD} 4f/2$$

$$h_f = \frac{4fLV^2}{2gD}$$

3. Using Buckingham's method, prove that the wave resi-stance of a ship is given by $V = \sqrt{gH}\ \phi\left[\frac{d}{H} \cdot \frac{\mu}{\rho V H}\right]$ where H = head of liquid, L = linear dimension, V = velocity of ship, ρ = density of liquid, ϕ = a functional notation.
where V = velocity through orifice ~~orifice~~ meter.

Functional relationship is given by

$$V = f[g, H, d, \mu, \rho]$$

Total no. of variables, $n = 6$

no. of π-terms required, $n - m = 3$

$V = m/s = LT^{-1}$, $\quad \mu = Ns/m^2 = ML^{-1}T^{-1}$

$H = m = L$, $\quad \rho = kg/m^3 = ML^{-3}$

$D = m = L$, $\quad g = m/s^2 = LT^{-2}$

φ or $f_1(\pi_1, \pi_2, \pi_3) = 0$

where H, g, ρ be the repeating variables

$$\pi_1 = H^{a_1} g^{b_1} \rho^{c_1} V$$

Substituting dimensions on both sides

$$M^0 L^0 T^0 = L^{a_1} (LT^{-2})^{b_1} (ML^{-3})^{c_1} (LT^{-1})$$

Equating the powers of M, L, T on both sides

powers of M, $\quad 0 = c_1$

powers of L, $\quad 0 = a_1 + b_1 - 3c_1 + 1, \quad 0 = a_1 - \frac{1}{2} + 1, \quad a_1 = -\frac{1}{2}$

powers of T, $\quad 0 = -2b_1 - 1, \quad b_1 = -\frac{1}{2}$.

$\Rightarrow$

$$\pi_1 = H^{-1/2} g^{-1/2} \rho^0 V$$

$$\pi_1 = \frac{V}{\sqrt{gH}}$$

Second, $\quad \pi_2 = H^{a_2} g^{b_2} \rho^{c_2} D$

Substituting dimensions on both sides

$$M^0 L^0 T^0 = L^{a_2} (LT^{-2})^{b_2} (ML^{-3})^{c_2} L$$

Equating the powers of M, L, T on both sides

for M, $\quad 0 = c_2$

for L, $\quad 0 = a_2 + b_2 - 3c_2 + 1, \quad \Rightarrow a_2 = -1$

for T, $\quad 0 = -2b_2 \quad b_2 = 0$

$\pi_2 = H^{-1} g^0 \rho^0 D$

$$\pi_2 = \frac{D}{H}$$

Third term, $\pi_3 = H^{a_3} g^{b_3} \rho^{c_3} \mu$

Substituting the dimensions on both sides.

$$M^0 L^0 T^0 = L^{a_3} (LT^{-2})^{b_3} (ML^{-3})^{c_3} \; ML^{-1}T^{-1}$$

Equating the powers of MLT on both sides

For M, $\quad 0 = c_3 + 1, \quad c_3 = -1$

For L, $\quad 0 = a_3 + b_3 - 3c_3 - 1$

For T, $\quad 0 = -2b_3 - 1, \quad b_3 = -1/2$

$$\Rightarrow \quad 0 = a_3 - \frac{1}{2} - 3(-1) - 1 = a_3 + \frac{3}{2}$$

$$a_3 = -3/2.$$

$$\pi_3 = H^{-3/2} g^{-1/2} \rho^{-1} \mu$$

$$\pi_3 = \frac{\mu}{\rho H^{3/2} \sqrt{g}}$$

$$\pi_3 = \frac{\mu}{\rho H \sqrt{gH}} = \frac{\mu}{\rho V H} \cdot \frac{V}{\sqrt{gH}} = \frac{\mu}{\rho V H} \cdot \pi_1$$

$$\Rightarrow \quad f_1 \left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \pi_1 \frac{\mu}{\rho V H} \right)$$

$$\Rightarrow \quad \frac{V}{\sqrt{gH}} = \phi \left[\frac{D}{H}, \frac{\mu}{\rho V H} \right]$$

$$V = \sqrt{gH} \; \phi \left[\frac{D}{H}, \frac{\mu}{\rho V H} \right]$$

MODEL OR SIMILARITY LAWS

For predicting the performance of the hydraulic structures (such as dams, spill ways etc) or hydraulic machines (such as turbines, pumps etc..), before actually constructing & manufacturing, models of the structures or machines are made and tests are performed on them to obtain the desired information.

The model is the small scale replica of the actual structure or machine. The actual structure or machine is called Prototype.

Similarity (Similitude)

Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties & model and prototype are completely similar. Three types of similarities must exist between the model and prototype. They are

1. Geometric Similarity:

The geometric similarity is said to exist between the model and the prototype if the ratios of all corresponding

Let L_m = Length of model, b_m = Breadth of model.

D_m = Diameter of model, A_m = Area of model.

V_m = volume of model

and L_p, B_p, D_p, A_p, V_p = corresponding values of the prototype.

$\Rightarrow$ For geometric similarity

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r \quad \text{, where } L_r = \text{Scale ratio.}$$

For Area's $\quad \frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r^2$

For volume, $\quad \frac{V_p}{V_m} = \frac{L_p \times b_p \times height_p}{L_m \times b_m \times height_m} = L_r \cdot L_r \cdot L_r = L_r^3$

2. <u>Kinematic Similarity:</u>

The kinematic similarity is said to exist between the model and the prototype if the ratios of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are same.

Velocity and acceleration are vector quantities hence both magnitude and direction of model and prototype should be same.

let V_{p_1} = Velocity of fluid at point 1 in prototype,

V_{p_2} = Velocity of fluid at point 2 in prototype

ap_1 = Acceleration of fluid at 1 in prototype.

ap_2 = Acceleration of fluid at point 2 in prototype.

and $V_{m_1}, V_{m_2}, a_{m_1}, a_{m_2}$ are corresponding values at the corresponding points of fluid velocity and acceleration in the model.

For kinematic similarity

$$\frac{V_{P_1}}{V_{m_1}} = \frac{V_{P_2}}{V_{m_2}} = V_r \quad , \quad V_r = \text{velocity ratio}$$

$$\frac{a_{m_1}}{a_{P_1}} = \frac{a_{m_2}}{a_{P_2}} = a_r \quad , \quad a_r = \text{acc. ratio}$$

3. Dynamic Similarity

Dynamic similarity is said to exist between the model and the prototype if the ratios of corresponding forces acting at the corresponding points are equal. Also directions of the corresponding forces at the corresponding points should be same.

let $(F_i)_p$ = Inertia force at a point in prototype

$(F_i)_m$ = Inertia force at a point in model

$(F_v)_p$ and $(F_g)_p$ are viscous force and gravity force at a point in prototype.

$(F_v)_m$ and $(F_g)_m$ are in model

For dynamic similarity.

$$\frac{(F_i)_P}{(F_i)_m} = \frac{(F_v)_P}{(F_v)_m} = \frac{(F_g)_P}{(F_g)_m} = F_r$$, F_r is the force ratio.

Model laws

1. Reynold's Model law

Reynold's number is the ratio of inertia force and viscous force. Reynolds model law is used to design models for dynamic similarity, where viscous forces alone are predominent. Reynolds model law states that " Reynold number for the model must be equal to the Reynold number for the prototype.

let V_m, ρ_m, μ_m and L_m be the velocity of fluid, density and viscosity of the fluids and length of the model in the model respectively.

V_p, ρ_p, μ_p and L_p are corresponding values of fluid in prototype.

$\Rightarrow$ $[Re]_m = [Re]_p$.

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

Example: i) Pipe flow

ii) Resistance experienced by sub marines, airplanes, fully immersed bodies

2. Froude's Model law.

Froude's model law states that "Froude number of the model should be equal to Froude number of the prototype". Froude number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity. Froude model law is applicable when the gravity force is only predominent.

Example: 1) Free surface flows such as over notch, weir channels etc...

2) where fluids of different densities flow over one anothes.

For Froudes Model law.

$$(Fe)_m = (Fe)_p.$$

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

where V = velocity of fluid
L = length

3. Euler model law

Euler number is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force. Euler model law states that "Euler number of prototype should be equal to Euler number of model". This law applicable when the pressure force are alone predominent.

ie $(Eu)_{model} = (Eu)_{prototype}$.

let V_m = Velocity of fluid in model.

P_m = Pressure of fluid in model.

ρ_m = Density of fluid in model.

and V_p, P_p, ρ_p are corresponding values in prototype,

$\Rightarrow$

$$\frac{V_m}{\sqrt{\frac{P_m}{\rho_m}}} = \frac{V_p}{\sqrt{\frac{P_p}{\rho_p}}}$$

ρ of fluid is same in model and prototype.

$$\Rightarrow \quad \frac{V_m}{\sqrt{P_m}} = \frac{V_p}{\sqrt{P_p}}$$

Example: Flow is taking place in a closed pipe in which case turbulence is fully developed so that viscous forces are negligible.

4. Weber's Model law

Weber model law is the law in which models are based on weber's number, which is the ratio of the square root of inertia force to surface tension force. Hence where surface tension effects predominate in addition to inertia force the dynamic similarity between the model and prototype is obtained by equaling the weber number of the model and its prototype.

$\Rightarrow$ $(We)_{model} = (We)_{prototype}$

let

V_m = velocity of fluid in model.

σ_m = Surface tensile force in model

ρ_m = Density of fluid in model

L_m = Length of surface in model

and $V_p, \sigma_p, \rho_p, L_p$ are corresponding values of prototype.

$\Rightarrow$

$$\frac{V_m}{\sqrt{\sigma_m / \rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p / \rho_p L_p}}$$

<u>Examples</u>: i) capillary rise in narrow passages

ii) capillary movement of water in soil.

5. Mach Model law

Mach model law is the law in which models are designed on Mach number, which is the ratio of the square root of inertia force to elastic force of a fluid. Hence where the forces due to elastic compression predominate in addition to inertia force, the dynamic similarity between the model and its prototype is obtained by equating the

Mach number of the model and its prototype

$\Rightarrow$

$$(M)_{model} = (M)_{prototype}$$

let V_m = Velocity of fluid in model

K_m = Elastic stress for model

ρ_m = Density of fluid in model.

and V_m, K_m, ρ_m are corresponding values for prototype.

$\Rightarrow$

$$\frac{V_m}{\sqrt{K_m/\rho_m}} = \frac{V_p}{\sqrt{K_p/\rho_p}}$$

Example: i) Flow of aeroplane

ii) Aerodynamic testing.

5. In a reservoir model built to a scale of 1:200, the rate of flow through the sluice into the canal is 2LPM and it takes 28.6 hours to drain the reservoir. Predict the prototype discharge and the time of emptying of the reservoir.

Froude's law of similitude is applicable in a free surface flow problem.

we have from Froude's law

$$\left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_p$$

if $g_p = g_m$.

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

$$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}.$$ length scale ratio $= L_r$.

$$V_r = \sqrt{L_r}.$$ V_r = velocity ~~rating~~ scale ratio

we have $Q = AV = L^2 V = L^2 \frac{L}{T}$

$$\frac{Q_p}{Q_m} = Q_r = \frac{(L_p^3/T_p)}{(L_m^3/T_m)} = \left(\frac{L_p}{L_m}\right)^3 \times \left(\frac{T_m}{T_p}\right) = L_r^3 \cdot \frac{1}{\sqrt{L_r}} \quad \text{[from ①]}$$

$$\frac{Q_p}{Q_m} = L_r^{2.5}$$

where, time $= \frac{\text{length}}{\text{velocity}}$, $T = \frac{L}{V}$.

$$\frac{T_p}{T_m} = T_r = \frac{(L_p/V_p)}{(L_m/V_m)} = \left(\frac{L_p}{L_m}\right) \cdot \left(\frac{V_m}{V_p}\right) = L_r \frac{1}{\sqrt{L_r}}.$$

$$T_r = \sqrt{L_r} \quad \text{— ①}$$

Given, $\frac{L_p}{L_m} = \frac{200}{1}$, $Q_m = 2\ lpm = \frac{2}{60} \times 10^{-3}\ m^3/s$

$T_m = 28.6$ hrs

To find $Q_p = ?$ $T_p = ?$

We know $Q_r = L_r^{2.5}$

$$\frac{Q_p}{Q_m} = \left(\frac{L_p}{L_m}\right)^{2.5}$$

$$Q_p = \frac{2}{60} \times 10^{-3} (200)^{2.5} = \underline{\underline{18.856 \ m^3/s}}$$

$$T_r = \sqrt{L_r}.$$

$$\frac{T_p}{T_m} = \sqrt{L_p/L_m} \Rightarrow T_p = T_m \sqrt{L_p/L_m}$$

$$= 28.6\sqrt{200} = \underline{\underline{404.465}} \text{ hours}$$

6. It is desired to obtain the dynamic similarity between a 30 cm diameter pipe carrying linseed oil at $0.5 m^3/s$ and a 5 cm diameter pipe carrying water. what should be the rate of flow of water in lpm ? If the pressure loss in the model is $196 N/m^2$, what is the pressure loss in the prototype pipe ? Kinematic ~~veloc~~ viscosities of linseed oil and water are 0.457 and 0.0113 stokes respectively. Specific gravity of linseed oil = 0.82

Reynolds law of similitude governs pipe flow modeling.

given

prototype | **model**

prototype	model
diameter of pipe 30cm $= d_p$	$d_m = 5cm = 0.05m$
c/s area $A_p = \frac{\pi}{4} 0.3^2$, linseed.	water.
$Q_p = 0.5 m^3/s$.	$\Delta p_m = 196 N/m^2$
we have $V_p = \frac{Q_p}{A_p}$.	$Q_m = ?$
$\Delta P_p = ?$	
$\nu_p = 0.457$ stoke.	$\nu_m = 0.0113$ stoke.
$S_p = 0.82$	$S_m = 1$

we have Reynold's model law.

$$\Rightarrow \quad \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

$$V_p = \frac{Q_p}{A_p} = \frac{0.5}{\frac{\pi}{4} \times 0.3^2} = 7.0735 \frac{m}{s}$$

ie $$\left(\frac{VD}{\nu}\right)_p = \left(\frac{VD}{\nu}\right)_m$$

$$\frac{7.0735 \times 0.3}{0.457 \times 10^{-4}} = \frac{V_m \times 0.05}{0.0113 \times 10^{-4}}$$

$$V_m = 1.0494 \; m/s$$

$$\therefore \; Q_m = V_m A_m = 1.0494 \times \frac{\pi}{4} \times 0.05^2 = \underline{\underline{2.06 \; lps}}$$

We know $\text{Pressure} = \dfrac{\text{Force}}{\text{Area of flow}}$.

We know Force = mass × acceleration

$$= \rho \forall \times \frac{V}{T} = \rho L^3 \frac{L}{T^2} = \rho L^2 V^2$$

ie $$F_r = \frac{F_P}{F_m} = \rho_r L_r^2 V_r^2$$

$$\text{Area} = L^2 \quad \text{ie } A_r = L_r^2 .$$

$$\text{pressure } (\Delta P)_r = \frac{\rho_r L_r^2 V_r^2}{L_r^2} = \rho_r V_r^2$$

$$(\Delta P)_P = (\Delta P)_m \frac{\rho_P}{\rho_m} \cdot \frac{V_P^2}{V_m^2} .$$

$$= 196 \times \frac{820}{1000} \times \frac{7.0735}{1.0494^2} = \underline{\underline{7302 \, N/m^2}} .$$

Model similarity of pumps and turbines

The complete similarity between the model and the prototype will exists if the following conditions are satisfied.

(1) $$\left[\frac{N\sqrt{Q}}{H^{3/4}}\right]_m = \left[\frac{N\sqrt{Q}}{H^{3/4}}\right]_P$$ for pump

(2) $$\left(\frac{H}{N^2 D^2}\right)_m = \left(\frac{H}{N^2 D^2}\right)_P$$

(3) $$\left[\frac{Q}{ND^3}\right]_m = \left[\frac{Q}{ND^3}\right]_p.$$

(4) $$\left[\frac{P}{N^3D^5}\right]_m = \left[\frac{P}{N^3D^5}\right]_p.$$

(5) $$\left[\frac{N\sqrt{P}}{H^{5/4}}\right]_m = \left[\frac{N\sqrt{P}}{H^{5/4}}\right]_p$$ for turbine

7. A model of a turbine built to a scale of 1:4 is tested under a head of 10m. The prototype has to work under a head of 50m at 450 RPM a) what speed should the model run be if it develops 60 kW using 0.9 m^3/s at this speed b) what power will be obtained from the prototype assuming that its efficiency is 3% better than that of model.

Given

model	prototype.
	Scale $\frac{D_p}{D_m} = 4$
$H_m = 10m$	$H_p = 50m$
$N_m = ?$	$N_p = 450 RPM$
$P_m = 60 kW$	$P_p = ?$
$Q_m = 0.9 m^3/s$	$\eta_p = 1.03\eta_m$

For geometrically similar machines

$$\left(\frac{H}{N^2D^2}\right)_m = \left(\frac{H}{N^2D^2}\right)_p$$

$$\frac{10}{N_m^2 \cdot 1^2} = \frac{50}{450^2 \times 4^2}$$

$$N_m = \underline{\underline{805 rpm}}$$

again we have.

$$\left(\frac{P}{\eta_m N^3 D^5}\right)_m = \left(\frac{P}{\eta N^3 D^5}\right)_p$$

given $\eta_p = 1.03 \eta_m$.

$$\frac{1.03 \times 60}{805^3 \times 1^5} = \frac{P_p}{450^3 \times 4^5}$$

$$P_p = \underline{\underline{11054.5 kW}}$$

8. A $\frac{1}{4}$th model of a centrifugal pump is tested at 3000rpm when it is delivering 10l/s of water at a head of 40m with an efficiency of 70%. Assuming that

the prototype gives an efficiency of 75% working against a head of 55 m, predict its speed, discharging capacity and ratio of power between the prototype and the model.

Given

$$\frac{D_m}{D_p} = \frac{1}{4}, \quad N_m = 3000\,rpm$$

$$Q_m = 10\,l/s = 0.01\,m^3/s, \; H_m = 40\,m, \; H_p = 55\,m$$

$$\eta_m = 0.7 \quad \eta_p = 0.75.$$

To find: N_p, Q_p, $P_p/P_m = ?$

we have.

$$\left(\frac{H}{N^2D^2}\right)_m = \left(\frac{H}{N^2D^2}\right)_p.$$

$$\frac{40}{3000^2 \times 1^2} = \frac{55}{N_p^2 \times 4^2}$$

$$N_p = \underline{\underline{879.45\,rpm}}$$

also
$$\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p$$

$$\frac{10}{3000 \times 1^3} = \frac{Q_p}{879.45 \times 4^3}$$

also we have

$$\left(\frac{P}{\eta N^3 D^5}\right)_m = \left(\frac{P}{\eta N^3 D^5}\right)_p .$$

$$\frac{P_p}{P_m} = \frac{\eta_p N_p^3 D_p^5}{\eta_m N_m^3 D_m^5}$$

$$= \frac{0.75 \times 879.45^{3} \times 4^{\cancel{8}5}}{0.7 \times 3000^{3} \times 1^{3}} .$$

$$\frac{P_p}{P_m} = \cancel{1.72405}\ \underline{\underline{27.64}}$$

9. A small scale model of Hydraulic turbine runs at a speed of 350 rpm under a head of 20 m and produces 8 kW as output assuming the turbine efficiency of 0.79, Find output power of the actual turbine which is 12 times the model size. assuming the model and prototype efficiency to be related by the Moody formula and type of runner you would use in this case.

Given $N_m = 350\,rpm$, $H_m = 20m$, $P_m = 8kW$ $\eta_m = 0.79$

To find Q_p, η_p. , $\frac{D_m}{D_p} = \frac{1}{12}$

We have

Moody's formula for Efficiency of prototype.

$$\eta_p = 1 - (1-\eta_m)\left(\frac{D_m}{D_p}\right)^{0.2}$$

$$= 1 - (1-0.79)\left(\frac{1}{12}\right)^{0.2}$$

$$\boxed{\eta_p = 0.8722}$$

we have,

assume $H_p = 12 H_m$

ie $H_p = 12 \times 20 = 240\,m$.

we have.

$$\left(\frac{P}{\eta N^3 D^5}\right)_m = \left(\frac{P}{\eta N^3 D^5}\right)_p.$$

but $\left(\frac{H}{N^2 D^2}\right)_m = \left(\frac{H}{N^2 D^2}\right)_p$

$$\frac{H_p}{H_m} = \frac{(N^2 D^2)_p}{(N^2 D^2)_m} \Rightarrow \left(\frac{H_p}{H_m}\right)^{3/2} = \frac{(ND)^3_p}{(ND)^3_m}$$

$$\frac{P_p}{P_m} = \frac{\eta_p}{\eta_m} \cdot \frac{D_p^2}{D_m^2}\left(\frac{H_p}{H_m}\right)^{3/2}$$

$$\frac{P_p}{8} = \frac{0.8722}{0.79} \cdot (12)^2 \cdot \left(\frac{240}{20}\right)^{3/2}$$

$P_p = 52810.5\,KW$

10. A model of Francis turbine of 1:5 scale ratio is tested under a head of 1.5m. It develops 3kw at 360rpm. Determine the speed and power developed under a head of 6m. Also find its specific speed.

Given

$$\frac{D_m}{D_p} = \frac{1}{5}, \quad H_m = 1.5m, \quad P_m = 3\,kw, \quad N_m = 360rpm$$

To find N_p, P_p $H_p = 6m$, N_s.

we have $$\left[\frac{H}{N^2D^2}\right]_m = \left[\frac{H}{N^2D^2}\right]_p.$$

$$\frac{1.5}{360^2 \times 1^2} = \frac{6}{N_p^2 \times 5^2}$$

$$N_p = \underline{144\,rpm}$$

Specific speed of model $$N_{s_m} = \frac{N_m\sqrt{P_m}}{H_m^{5/4}}.$$

$$= \frac{360\sqrt{3}}{1.5^{5/4}} = \underline{\underline{315.62}}$$

we have.

$$\left(\frac{P}{N^3 D^5}\right)_m = \left(\frac{P}{N^3 D^5}\right)_p$$

$$\frac{P_P}{P_m} = \left(\frac{H_P}{H_m}\right)^{3/2}\left(\frac{D_P}{D_m}\right)^2$$

$$\frac{P_p}{3} = \left(\frac{6}{1.5}\right)^{3/2} \cdot \left(\frac{5}{1}\right)^2$$

$$P_p = 600\,kW$$

Specific speed of prototype.

$$N_{sp} = \frac{N_p\sqrt{P_P}}{H_p^{5/4}} = \frac{144\sqrt{600}}{6^{5/4}}$$

$$N_s = 375.62$$

11. A model of a centrifugal pump, absorb 5kW at a speed of 1500 rpm pumping water against a head of 6m. The large prototype pump is required to pump water to a head of 30m. The scale ratio of diameter is 4. Assuming same efficiency and similarity. Find the speed and power of the prototype and also the ratio of discharge of prototype and the model.

Given,

$P_m = 5\,kW$, $N_m = 1500\,rpm$. $H_m = 6m$.

$H_p = 30m$, $\frac{D_P}{D_m} = 4$.

$\eta_m = \eta_P$

To find. N_P, P_P, $\frac{Q_P}{Q_m} = ?$

we have

$$\left[\frac{H}{N^2D^2}\right]_m = \left[\frac{H}{N^2D^2}\right]_m .$$

$$\frac{6}{1500^2 \times 1} = \frac{30}{N_P^2 \times 4^2} .$$

$$N_P = \underline{\underline{838.53\,rpm}}$$

we have.

$$\left[\frac{P}{N^3D^5\eta}\right]_m = \left[\frac{P}{\eta D^5 N^3}\right]_P .$$

$$\frac{5}{1500^3 \times 1 \times 1} = \frac{P_P}{1 \times 4^5 \times 838.53^3} .$$

$$P_P = \underline{\underline{894.43\,kW}}$$

again $$\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_P .$$

$$\frac{Q_P}{Q_m} = \frac{838.53 \times 4^3}{1500 \times 1^3} = \underline{\underline{35.777}}$$

12. A model operates under a head of 5m at 1200rpm. The power in the laboratory is limited to 8kW. Predict the power and the diameter ratio of a prototype turbine which operates under a head of 40m at 240 rpm. What type of turbine is the prototype? Pelton, Francis & Kaplan.

Given

model	prototype
$H_m = 5m$	$P_p = ?$, $D_p/D_m = ?$
$N_m = 1200 rpm$	$H_p = 40m$
$P_m = 8kW$	$N_p = 240 rpm$.

we have

$$\left[\frac{H}{N^2D^2}\right]_m = \left[\frac{H}{N^2D^2}\right]_p$$

$$\frac{5}{1200^2 \times D_m^2} = \frac{40}{240^2 \times D_p^2}$$

$$\frac{D_p}{D_m} = \underline{\underline{14.14}}$$

again

$$\left[\frac{P}{N^3D^5}\right]_m = \left[\frac{P}{N^3D^5}\right]_p$$

$$\frac{8}{1200^3 \times D_m^5} = \frac{P_p}{240^3 \times D_p^5}$$

$$P_p = \frac{8 \times 14.14 \times 240}{1200^3}$$

$$= 36203.87 \text{ kW}$$

$$P_p = \underline{\underline{36.2 \text{ MW}}}$$

also

NB. Specific Speed $N_s = \dfrac{N_p \sqrt{P_p}}{H_p^{5/4}}$

$$= \frac{240 \sqrt{36203.87}}{40^{5/4}}$$

$$= 453.96 \simeq \underline{\underline{454}} \quad \Rightarrow \text{Kaplan turbine.}$$

ENERGY EXCHANGE and ANALYSIS OF TURBOMACHINES.

The fluid flow through the turbomachine rotor is assumed to be steady and mass flow is constant, also rates of energy transfer is constant. It is assumed that losses due to leakage are negligible and that the same steady mass of fluid flows through all the sections.

The absolute velocity of the fluid entering into the turbomachine is resolved into three components

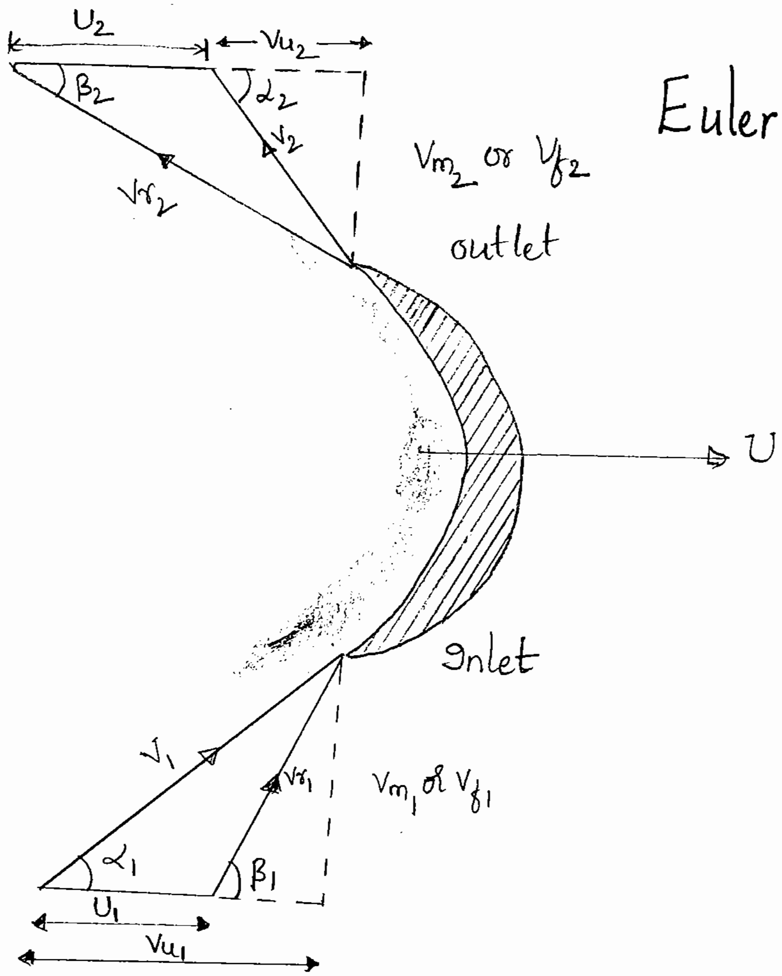
(a) Axial component (V_a) which is along the axis of rotation.

(b) Radial component (V_{rd} & V_m) which is $\perp^{er}$ to the axis of rotation.

(c) Tangential component (V_u) which is along the tangential direction of the rotor.

The change in magnitude of axial velocity components give rise to a axial thrust which must be taken up by the thrust bearings. The change in magnitude of radial velocity components give rise to a radial thrust which is to be taken up by the

Journal bearing. Neither of these forces cause no angular rotation nor has any effect on the torque exerted on the rotor. The only velocity component which changes the angular momentum of fluid is the tangential component and by Newton's law of motion which is equal to the summation of all the applied forces on the rotor (ie net torque).



Euler turbine Equation

Let V_1 be the absolute velocity of the fluid.

U_1 be the tangential speed of rotation of rotor at inlet

(or) linear velocity of the rotor at inlet

(or) Velocity of vane at inlet.

V_{r_1} be the relative velocity at inlet.

V_{u_1} be the tangential component of v_1 at inlet

(or)

The component of v_1 in the direction of rotor.

(or) Velocity of whirl at inlet.

V_{m_1} be the component of v_1 in the direction perpendicular to the motion (or) Meredonial component of absolute flow of fluid.

α_1 be the Nozzle angle or fixed blade angle at inlet

β_1 be the blade angle at inlet [moving blade]

<u>and</u>

V_2, U_2, V_{r_2}, V_{u_2}, V_{m_2}, α_2 and β_2 are the corresponding values of absolute velocity, vane velocity, relative velocity, whirl velocity, axial component of absolute velocity, fixed blade angle and blade angle at outlet.

The fluid is striking on vane at absolute velocity v_1 and resulting in the rotation of rotor. The linear velocity of rotor at inlet and outlet are not equal if radius of rotor at inlet and outlet are different.

ie $U_1 \neq U_2$

also $U_1 = \omega R_1$ and $U_2 = \omega R_2$

where ω is angular velocity. The mass of fluid which is striking on the vane at inlet is

$$\dot{m} = \rho A V_1 \qquad \text{ie mass} = \text{Density} \times \text{volume}$$

where ρ = density of fluid in kg/m^3

A = cross sectional area in m^2

V_1 = Absolute velocity of fluid at inlet in m/s.

As we assume constant flow, mass of fluid leaving the rotor is same (ie $\rho A V_1$).

Momentum of fluid striking the vane/sec in the tangential direction at inlet

= (mass of fluid/sec) x Component of V_1 in tangential dirn

$$= \rho A V_1 \times V_{u_1} \quad \text{— (a)}$$

Simillarly at outlet.

$$= \rho A V_1 \times V_{u_2} \quad \text{— (b)}$$

Angular momentum of fluid at inlet/sec is

$$= \rho A V_1 V_{u_1} \cdot R_1$$

and at outlet/sec is

$$= \rho A V_1 V_{u_2} R_2 .$$

From Newton's law,

Torque exerted by fluid on the rotor = Rate of change of angular momentum.

$$= \rho A V_1 V_{u_1} R_1 - \rho A V_1 V_{u_2} R_2$$

$$T = \rho A V_1 [V_{u_1} R_1 - V_{u_2} R_2] \quad \text{—— (1)}$$

Work done by the fluid on the rotor is

work done (W) = Torque (T) x angular velocity

$$W = T \times \omega$$

$$= \rho A V_1 [V_{u_1} \omega R_1 - V_{u_2} \omega R_2]$$

but $\omega R_1 = U_1$ and $\omega R_2 = U_2$, Since $\omega = \frac{2\pi N}{60}$.

$$U = \frac{\pi D N}{60}.$$

ie $$W = \rho A V_1 [V_{u_1} U_1 - V_{u_2} U_2] \quad \text{—— (2)}.$$

Energy transfer in turbomachine is workdone per unit mass flow rate.

ie $$E = \frac{W}{\dot{m}} = \frac{\rho A V_1 (V_{u_1} U_1 - V_{u_2} U_2]}{\rho A V_1}$$

$$E = U_1 V_{u_1} - U_2 V_{u_2}$$

Introducing the unit conversion factor $g_c = 1$.

$$\boxed{E = \frac{U_1 V_{u_1} - U_2 V_{u_2}}{g_c}}$$

power developed or absorbed by the turbomachine is

$$P = \text{mass} \times \text{energy transfer}$$

$$\boxed{P = mE} \quad \text{or} \quad P = \frac{m\left[V_{u_1}U_1 - U_2V_{u_2}\right]}{g_c}$$

Case

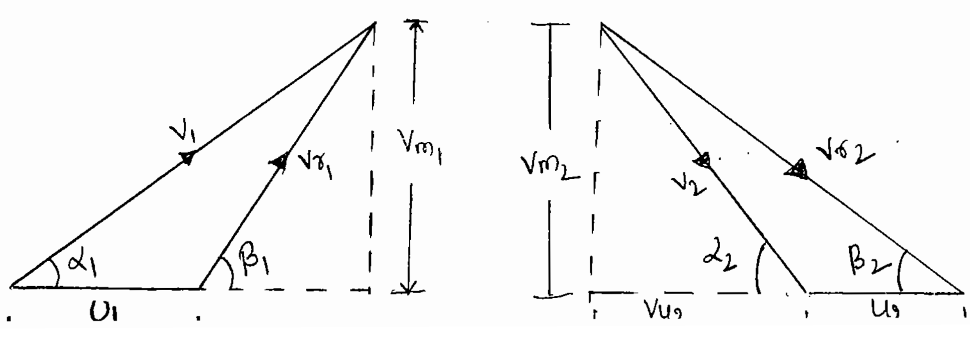
1) If $V_{u_1}U_1 > U_2V_{u_2}$

Power 'p' is +ve, hence power is produced, it is a power generating turbomachine [Turbines]

2) If $V_{u_1}U_1 < U_2V_{u_2}$

Power 'p' is −ve, hence power is absorbed, it is a power absorbing turbomachine (eg pumps, compressor).

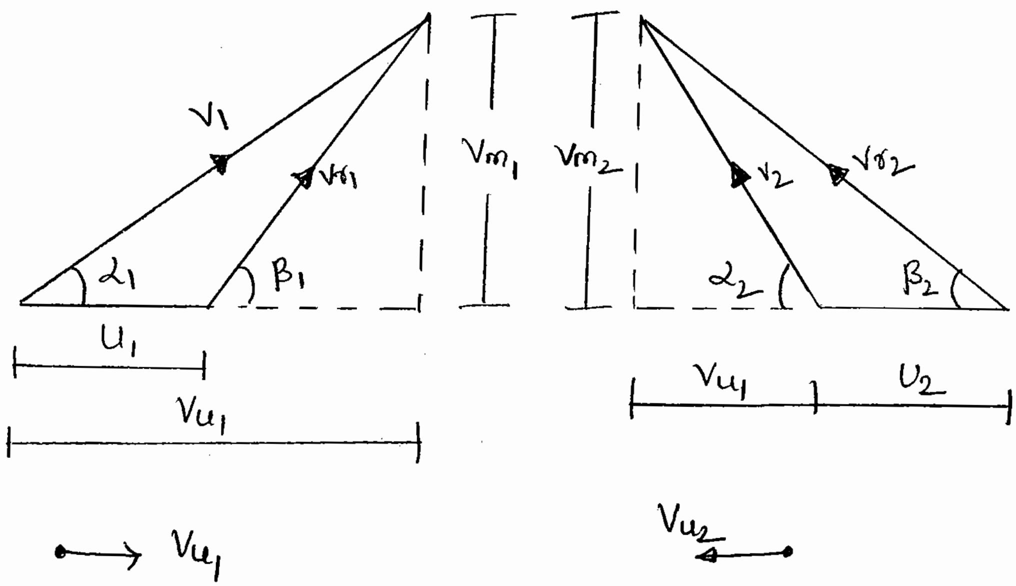
3) Velocity triangle for inward flow. [It means fluid enters at outer radius and leaves at inner radius].



Since both have same direction.

$$P = \dot{m}\left[\frac{V_{u_1}U_1 - U_2 V_{u_2}}{g_c}\right]$$

4) Outer flow velocity triangles (fluid enters at inner and leaves at outer radius of the rotor, ie



Hence, power $P = m\left[\frac{V_{u_1}U_1 + U_2 V_{u_2}}{g_c}\right]$

1. A turbomachine has inner and outer radius of 8 cm and 15 cm. The fluid enters at inner radius and leaves at larger radius of wheel. The fluid enters the wheel at an angle of 22° with a velocity of flow is 43 m/s. The The absolute velocity of fluid at rotor exit is 16 m/s and direction of 36° from the wheel tangent. The speed of rotor

is 3000rpm. Draw the velocity triangle and find

a) power output if mass flow rate of fluid is 10kg/s.

b) Relative velocity at inlet and outlet

c) Blade angles, Merodinal components of the absolute velocity at inlet and outlet

d) Energy transfer, Tangential components at inlet & o/L.

Given

Outward flow turbomachine

inner radius $R_1 = 08\,cm$, $D_1 = 16\,cm = 0.16\,m$

outer radius, $R_2 = 15\,cm$, $D_2 = 30\,cm = 0.3\,m$

Nozzle angle $\alpha_1 = 22°$ at inlet

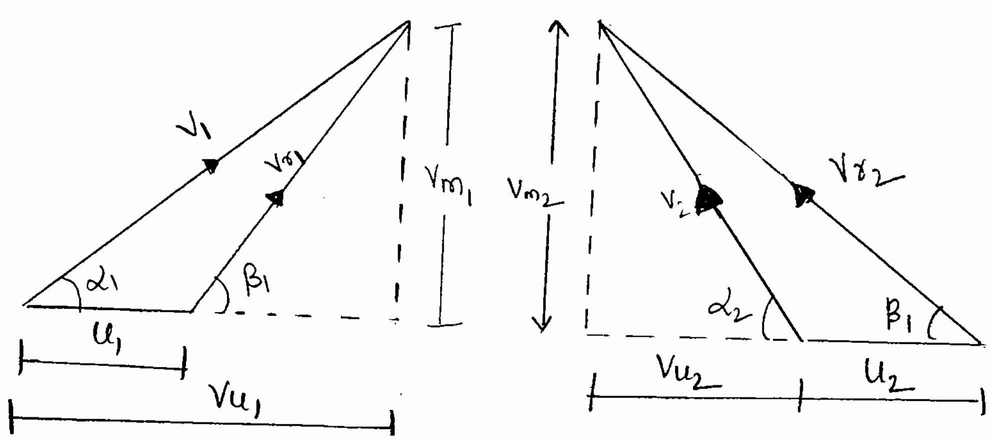
Absolute velocity $V_1 = 43\,m/s$. at inlet

Absolute velocity $V_2 = 16\,m/s$ at outlet.

Fixed blade angle $\alpha_2 = 36°$ at outlet.

Speed $N = 3000\,rpm$

To find. E, P, V_{r_1}, V_{r_2}, β_1, β_2, V_{m_1}, V_{m_2}, V_{u_1} and V_{u_2}.



We know,

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.16 \times 3000}{60} = 25.133 \text{ m/s}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 3000}{60} = 47.124 \text{ m/s}.$$

From velocity triangles,

at inlet

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$\cos 22 = \frac{V_{u_1}}{43}$$

$$\boxed{V_{u_1} = 39.869 \text{ m/s}}$$

at outlet

$$\cos\alpha_2 = \frac{V_{u_2}}{V_2}$$

$$\cos 36 = \frac{V_{u_2}}{16}$$

$$\boxed{V_{u_2} = 12.944 \text{ m/s}}$$

here $V_{u_1} \rightarrow$

$V_{u_2} \leftarrow$

here Energy $E = \dfrac{U_1 V_{u_1} + V_{u_2} U_2}{g_c}$

$$= \frac{(25.133 \times 39.869) + (12.944 \times 47.124)}{1}$$

$$\boxed{E = 1612 \text{ J/kg}}$$

also power $P = \dot{m} E = 10 \times 1612.$

$P = 16120 \text{ J/s}$

$$\boxed{P = 16.12 \text{ kW}}$$

From velocity triangles.

at inlet

$$\sin\alpha_1 = \frac{V_{m_1}}{V_1}$$

$$\sin 22 = \frac{V_{m_1}}{43}$$

$$\boxed{V_{m_1} = 16.108\,m/s}$$

$$\cos\beta_1 = \frac{V_{u_1} - u_1}{V_{r_1}}$$

but $\tan\beta_1 = \dfrac{V_{m_1}}{V_{u_1} - u_1}$

$$\tan\beta_1 = \frac{16.108}{39.869 - 25.133}$$

$$\beta_1 = \underline{\underline{47.55^\circ}}$$

ie $\cos\beta_1 = \dfrac{V_{u_1} - u_1}{V_{r_1}}$

$$V_{r_1} = \frac{39.869 - 25.133}{\cos 47.55}$$

$$\boxed{V_{r_1} = 21.833\,m/s}$$

at outlet

$$\sin\alpha_2 = \frac{V_{m_2}}{V_2}.$$

$$\sin 36 = \frac{V_{m_2}}{16}$$

$$\boxed{V_{m_2} = 9.405\,m/s}$$

$$\tan\beta_2 = \frac{V_{m_2}}{u_2 + V_{u_2}}$$

$$= \frac{9.405}{41.124 + 12.944}$$

$$\beta_2 = 8.899 \simeq \underline{\underline{8.9^\circ}}$$

$$\sin\beta_2 = \frac{V_{m_2}}{V_{r_2}}.$$

$$V_{r_2} = \frac{9.405}{\sin 8.9}$$

$$\boxed{V_{r_2} = 60.8\,m/s}$$

Sp. case. If fluid enters outer radius and leaves at inner radius (inward flow)

$$\Rightarrow \quad E = \left[\frac{V_{u_1}U_1 - V_{u_2}U_2}{g_c}\right]$$

here $U_1 \Rightarrow U_2$

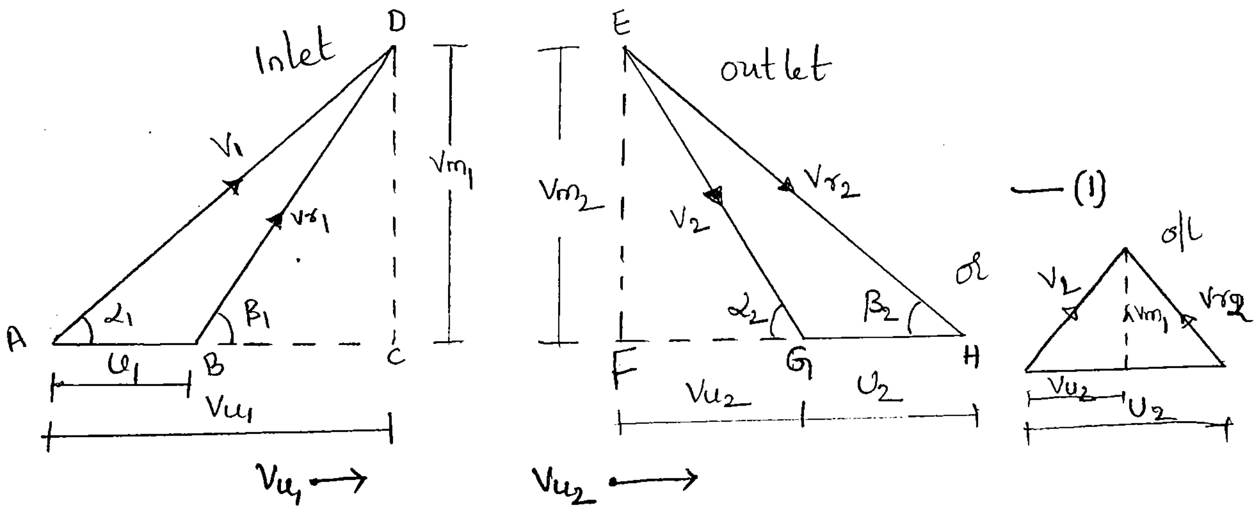
$U_2 \rightarrow U_1$

$E = [39.869 \times 47.124 - 12.944 \times 25.133]$

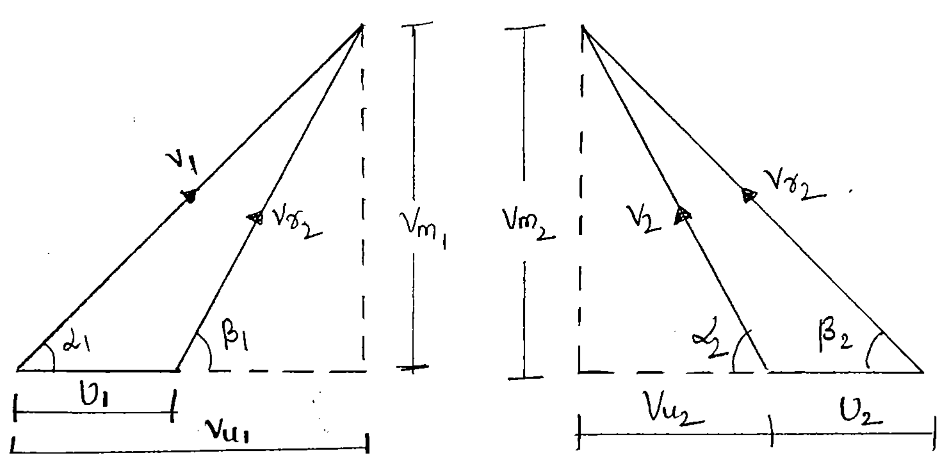
$E = 1553.47$ J/kg & $P = 15.53$ kW

(Inlet velocity Δle changes)

Alternative form of Energy Equation [Euler equation]



— (1)



— (2)

Let us consider inlet and outlet velocity Δles.

also Euler turbine equation $E = \frac{[U_1 V_{u_1} - U_2 V_{u_2}]}{g_c}$ — (1)

From inlet velocity triangle,

consider Δ ACD

$$CD^2 = AD^2 - AC^2$$

$$V_{m_1}^2 = V_1^2 - V_{u_1}^2 \quad \text{— (a)}$$

consider Δ BCD,

$$CD^2 = BD^2 - BC^2$$

$$V_{m_1}^2 = V_{r_1}^2 - (V_{u_1} - U_1)^2 \quad \text{— (b)}$$

Since (a) and (b) are equal.

$$V_1^2 - V_{u_1}^2 = V_{r_1}^2 - (V_{u_1} - U_1)^2$$

$$V_1^2 - V_{u_1}^2 = V_{r_1}^2 - V_{u_1}^2 - U_1^2 + 2U_1 V_{u_1}$$

$$V_1^2 + U_1^2 - V_{r_1}^2 = 2U_1 V_{u_1}$$

$$U_1 V_{u_1} = \frac{1}{2}\left[V_1^2 + U_1^2 - V_{r_1}^2\right] \quad \text{— (2)}$$

Simillarly from outlet velocity triangle

$$U_2 V_{u_2} = \frac{1}{2}\left[V_2^2 + U_2^2 - V_{r_2}^2\right] \quad \text{— (3)}$$

substituting equation (2) and (3) in (1).

$$(1) \Rightarrow \quad E = \frac{1}{g_c}\left[\frac{1}{2}\left(V_1^2 + U_1^2 - V_{r_1}^2\right) - \frac{1}{2}\left(V_2^2 + U_2^2 - V_{r_2}^2\right)\right]$$

$$E = \frac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{2g_c}$$

This is the alternative form of Euler turbine or Energy equation.

The pair of terms in each bracket indicates the nature of energy transfer.

(1) $\frac{(V_1^2 - V_2^2)}{2}$ is the change in absolute kinetic energy of the fluid between the inlet and outlet.

(2) $\frac{(U_1^2 - U_2^2)}{2}$ is the change in fluid energy due to a one radius to the other.

(3) $\frac{(V_{r_2}^2 - V_{r_1}^2)}{2}$ is the kinetic energy change due to a change of relative velocity between the inlet and exit of the rotor.

(1) term is called dynamic head and (2) and (3) are together called static head.

Degree of Reaction (R)

The degree of reaction is defined as "the ratio of change in static pressure in the rotor to the total energy ie

$$R = \frac{\text{change in static head in rotor}}{\text{Total Energy transfer in rotor}} = \frac{\Delta h_s}{\Delta h_o}$$

$$R = \frac{\frac{1}{2g_c}\left[(U_1^2-U_2^2)+(V_{r_2}^2-V_{r_1}^2)\right]}{\frac{1}{2g_c}\left[(V_1^2-V_2^2)+(U_1^2-U_2^2)+(V_{r_2}^2-V_{r_1}^2)\right]}$$

$$\boxed{R = \frac{(U_1^2-U_2^2)+(V_{r_2}^2-V_{r_1}^2)}{(V_1^2-V_2^2)+(U_1^2-U_2^2)+(V_{r_2}^2-V_{r_1}^2)}}$$

Hence degree of reaction is also defined as "ratio of static head due to change in static pressure to the total head due to change in total pressure in a rotor".

ie $\quad R = \frac{S}{D+S}$ where, S = Static head

D = Dynamic head.

$$\frac{1}{R} = \frac{D+S}{S} = \frac{D}{S}+\frac{S}{S}$$

$$\frac{1}{R} = \frac{D}{S}+1$$

$$\frac{1-R}{R} = \frac{D}{S}$$

$$\boxed{D = \frac{S(1-R)}{R}} \quad \text{or} \quad \boxed{S = D\left(\frac{R}{1-R}\right)}$$

Degree of reaction is also defined as "ratio of static

Case 1: Impulse machine

An impulse stage is one in which the static pressure at the rotor inlet is the same as that at the rotor outlet. often, the impulse stage is defined as one where the relative velocity of fluid flow is constant on the rotor.

ie change in static head is zero

ie $S = 0$

ie $R = \frac{S}{D+S}$, $\Rightarrow R = 0$

Hence Degree of reaction is '0' for impulse machines.

Case 2: Reaction machine

A reaction stage is one where a change in static pressure occurs during flow over each rotor stage.

a) For ideal, $R = 1$

ie $R = \frac{S}{D+S}$

$D+S = S$

$D = 0$, ie $(V_1^2 - V_2^2) = 0$

$\Rightarrow$ $V_1 = V_2$

ie, Absolute velocity of fluid at inlet is equal to absolute velocity of fluid at outlet. Generally it is not possible.

b) When $R > 1$

we have $R = \dfrac{S}{D+S}$

$$\Rightarrow \quad 1 < \frac{S}{D+S}$$

$$D+S < S$$

$$D < 0$$

$$(V_1^2 - V_2^2) < 0$$

$$\boxed{V_1 < V_2}$$

$\Rightarrow$ Power absorbing machine. [R is +ve]

c) when $R < 0$

we have $R = \dfrac{S}{D+S}.$

$$0 > \frac{S}{D+S}$$

$$0 \times (D+S) > S$$

$$0 > S, \qquad 0 > V_{r_2}^2 - V_{r_1}^2$$

$$\Rightarrow V_{r_1} > V_{r_2}$$

$\Rightarrow$ Power producing machine [R is -ve]

d) 50% Reaction turbomachine

ie $R = 0.5 = \frac{1}{2}$

we have $R = \frac{S}{D+S}$

$$\frac{1}{2} = \frac{S}{D+S}$$

$$\frac{D}{2} + \frac{S}{2} = S, \qquad \frac{D}{2} = \frac{S}{2}.$$

ie $(V_1^2 - V_2^2) = (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)$

if $U_1 = U_2$.

$$(V_1^2 - V_2^2) = (V_{r_2}^2 - V_{r_1}^2)$$

$$\Rightarrow \boxed{\begin{array}{c} V_1 = V_{r_2} \\ \hline V_2 = V_{r_1} \end{array}}$$

ie Absolute velocity of fluid at inlet (V_1) is equal to relative velocity at outlet (V_{r_2}) and absolute velocity of fluid at outlet (V_2) is equal to relative velocity at inlet (V_{r_1}).

Utilization factor (ϵ).

The utilization factor is the "ratio of the ideal work output to the energy available for conversion into work". It is also called diagram efficiency or blade efficiency.

For the rotor, the energy supplied is given by.

$$E_{supplied} = \frac{1}{2}\left[V_1^2 + (V_{r_2}^2 - V_{r_1}^2) + (U_1^2 - U_2^2) \right] \quad \text{—} (1)$$

The energy utilized by the rotor (from Euler equation) is given by

$$E_{utilized} = \frac{1}{2}\left[(V_1^2 - V_2^2) + (V_{r_2}^2 - V_{r_1}^2) + (U_1^2 - U_2^2) \right] \quad \text{—} (2)$$

According to the definition, utilization factor is given by

$$\epsilon = \cancel{\frac{E_{supplied}}{E}} = \frac{E_{utilized}}{E_{supplied}} = \frac{E_{utilized}}{E_{utilized} + \text{losses}}$$

$$\boxed{\epsilon = \frac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{V_1^2 + (V_{r_2}^2 - V_{r_1}^2) + (U_1^2 - U_2^2)} \quad \text{or} \quad \epsilon = \frac{E}{E + \frac{V_2^2}{2}}}$$

<u>Relation between (ε) utilization factor and Degree of Reaction (R)</u>

We know, degree of reaction $R = \frac{S}{D+S}$

$$\text{or} \quad \frac{1}{R} = \frac{D}{S} + 1$$

$$\frac{1-R}{R} = \frac{D}{S}$$

$$S = \left(\frac{R}{1-R}\right) D$$

ie $$(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2) = \left(\frac{R}{1-R}\right)(V_1^2 - V_2^2) \quad \text{—} (1)$$

We know,

$$\epsilon = \frac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{V_1^2 + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}$$

$$\epsilon = \frac{(V_1^2 - V_2^2) + \left(\frac{R}{1-R}\right)(V_1^2 - V_2^2)}{V_1^2 + \left(\frac{R}{1-R}\right)(V_1^2 - V_2^2)} \qquad \text{from (1)}$$

$$\epsilon = \frac{(1-R)(V_1^2 - V_2^2) + R(V_1^2 - V_2^2)}{(1-R)V_1^2 + R(V_1^2 - V_2^2)}$$

$$= \frac{(V_1^2 - V_2^2) - R(V_1^2 - V_2^2) + R(V_1^2 - V_2^2)}{V_1^2 - RV_1^2 + RV_1^2 - RV_2^2}$$

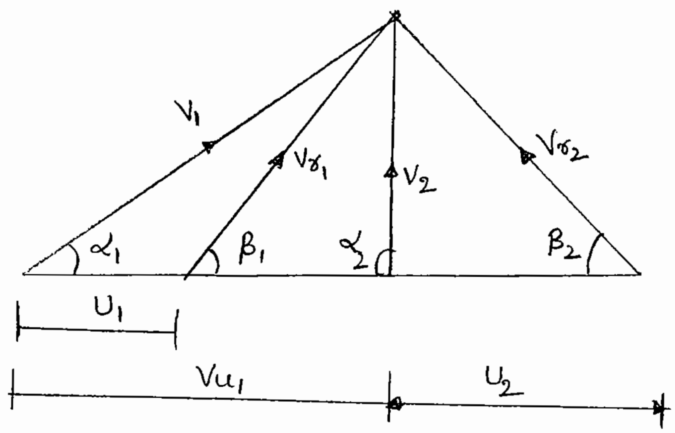
$$\boxed{\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}}$$

Maximum utilization factor

We know, $\epsilon = \dfrac{E}{E + \frac{V_2^2}{2}}$

For maximum utilization factor, the value of V_2 should be minimum and from the velocity triangle, it is apparent that V_2 is having minimum value when it is axial. Thus a general velocity diagram for maximum

Utilization is,



Here $V_2 = V_{m_1} = V_{m_2}$

$\alpha_2 = 90^\circ$

We have, $$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - R V_2^2} \quad \text{—— (1)}$$

From velocity triangle, $\sin\alpha_1 = \dfrac{V_2}{V_1}$

$$V_2 = V_1 \sin\alpha_1 \quad \text{—— (a)}$$

put (a) in (1)

$$\Rightarrow \quad \epsilon_{max} = \frac{V_1^2 - V_1^2 \sin^2\alpha_1}{V_1^2 - R V_1^2 \sin^2\alpha_1}$$

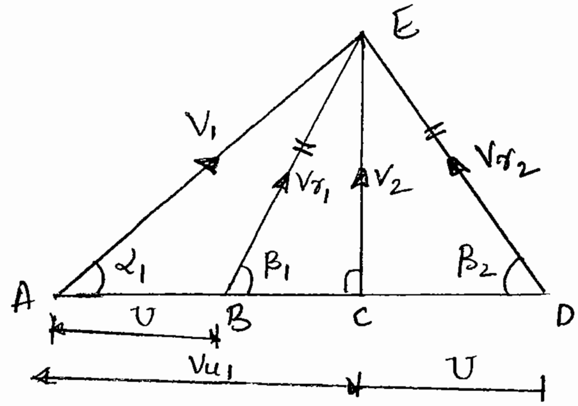
$$= \frac{V_1^2(1 - \sin^2\alpha_1)}{V_1^2(1 - R\sin^2\alpha_1)}$$

$$\boxed{\epsilon_{max} = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}}$$

Utilization factor is maximum (ie unity), when $\alpha_1 = 0$

a) Impulse turbine (condition for ϵ_{max})

Velocity Δ^{le} for impulse turbine is as shown below. For the impulse type, $R=0$



$U_1 = U_2 = U$

We have $\quad \epsilon = \dfrac{V_1^2 - V_2^2}{V_1^2 - R V_2^2}$ and $\epsilon_{max} = \dfrac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}$

but $R=0$ for impulse type.

$$\epsilon_{max} = \frac{\cos^2\alpha_1}{1 - 0\sin^2\alpha_1}$$

$$\boxed{\epsilon_{max} = \cos^2\alpha_1}$$

again,

Energy transfer, $E = \left(\dfrac{U_1 V_{u_1} - U_2 V_{u_2}}{g_c}\right)$ — (2)

~~At~~ here, $U_1 = U_2 = U$

$V_{u_2} = 0$, since V_2 is $\perp^{er}$

also for impulse $V_{r_1} = V_{r_2}$ & $\beta_1 = \beta_2$

$\Rightarrow \quad AC = AB + BC = AB + CD$

$V_{u_1} = U + U = 2U$

(2) $\Rightarrow$ $$E = \frac{U(2U)}{g_c} = \frac{2U^2}{g_c} \text{ or}$$

$$\boxed{E = 2U^2}$$

again speed ratio $\phi = \frac{u}{V_1}$

from velocity Δ^{le}, $\cos\alpha_1 = \frac{V_{u1}}{V_1}$

$$\cos\alpha_1 = \frac{2U}{V_1}$$

$$\frac{\cos\alpha_1}{2} = \frac{U}{V_1}$$

Hence speed ratio $\phi = \frac{\cos\alpha_1}{2}$

b) 50% Reaction turbine

For 50% reaction turbine, $R = 0.5$

we have $E_{max} = \frac{\cos^2\alpha_1}{1 - R\sin^2\alpha_1}$

ie $$E_{max} = \frac{\cos^2\alpha_1}{1 - 0.5\sin^2\alpha_1} = \frac{\cos^2\alpha_1}{\left(\frac{2 - \sin^2\alpha_1}{2}\right)}$$

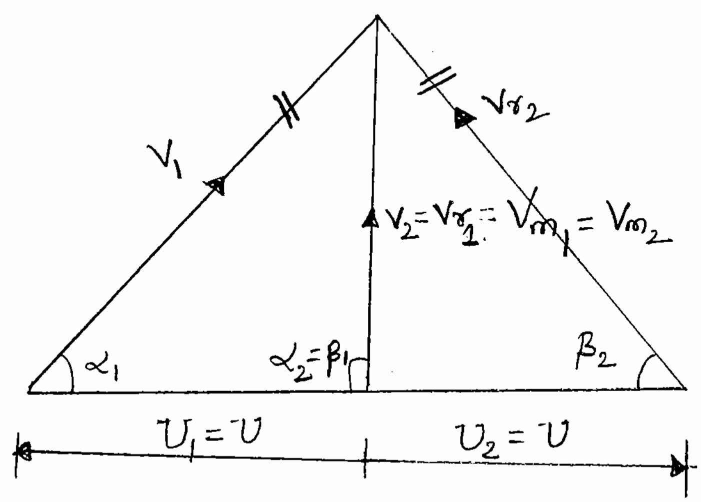
$$E_{max} = \frac{2\cos^2\alpha_1}{1 + 1 - \sin^2\alpha_1} = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1}$$

$$\boxed{E_{max} = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1}}$$

also, $V_1 = V_{r_2}$, $V_2 = V_{r_1}$

and for max. utilization factor V_2 should be minimum, that is, V_2 is Perpendicular.

ie $\Rightarrow$



also, Energy transfer, $E = \dfrac{U_1 V_{u_1} - U_2 V_{u_2}}{g_c}$.

but $V_{u_1} = U$, $U_1 = U_2 = U$,

$V_{u_2} = 0$

$$\Rightarrow \quad E = \frac{U \cdot U - U \cdot 0}{g_c} = U^2.$$

$$\boxed{E = U^2}$$

Speed ratio, $\phi = \dfrac{U}{V_1}$

again, $\cos \alpha_1 = \dfrac{U}{V_1}$

$$\boxed{\phi = \cos\alpha_1}$$

Hints for solving problems [To draw vel. Δ^{les}]

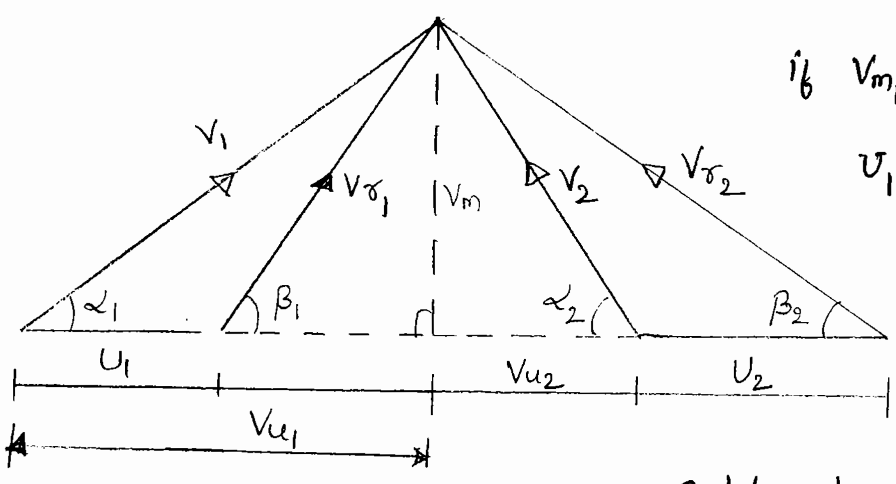
1. If the machine is of axial flow type,
   ie Fluid enters at and leaves at same radius.

   then $R_1 = R_2$, $U_1 = U_2 = U = \frac{\pi D N}{60} = \ldots$ m/s

2. For radial flow turbomachines.
   The fluid enters at radius r_1 and leaves at radius r_2, then $U_1 \neq U_2$

3. If the machine is of impulse type,
   $R = 0$, $U_1 = U_2 = U$, $V_{r_1} = V_{r_2}$ (unless specified)

4. If the machine is of 50% Reaction.
   $R = 0.5$, $V_1 = V_{r_2}$ $(\alpha_1 = \beta_2)$, $V_2 = V_{r_1}$ $(\alpha_2 = \beta_1)$



if $V_{m_1} = V_{m_2} = V_m$,

$U_1 = U_2 = U$,

Inlet and outlet vel. Δ^{les} are symmetric

5. For inward flow turbomachine. (Ex: Francis turbine)

The fluid enters at larger radius or outer radius and leaves at inner radius.

ie $R_1 > R_2 \Rightarrow U_1 > U_2$

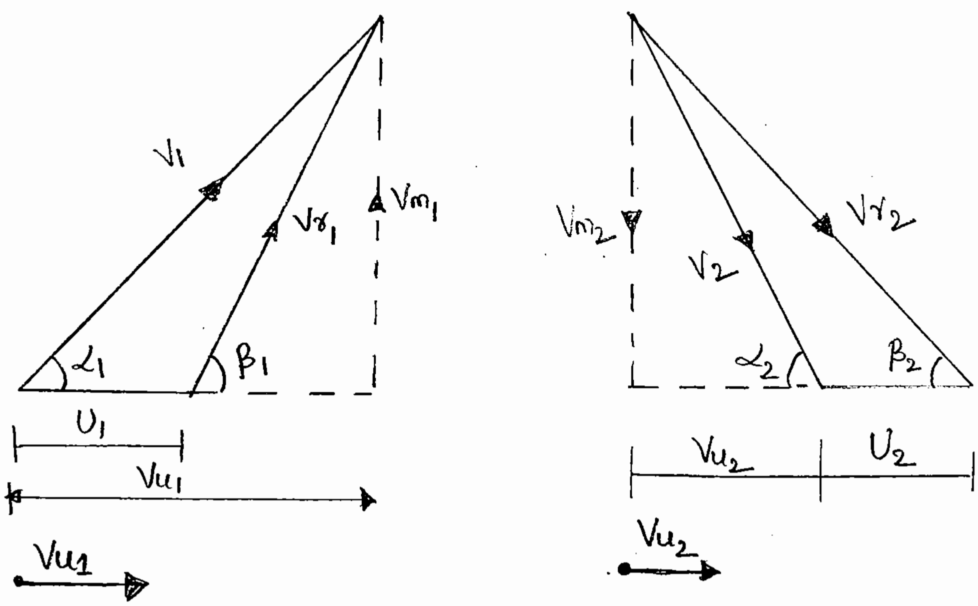
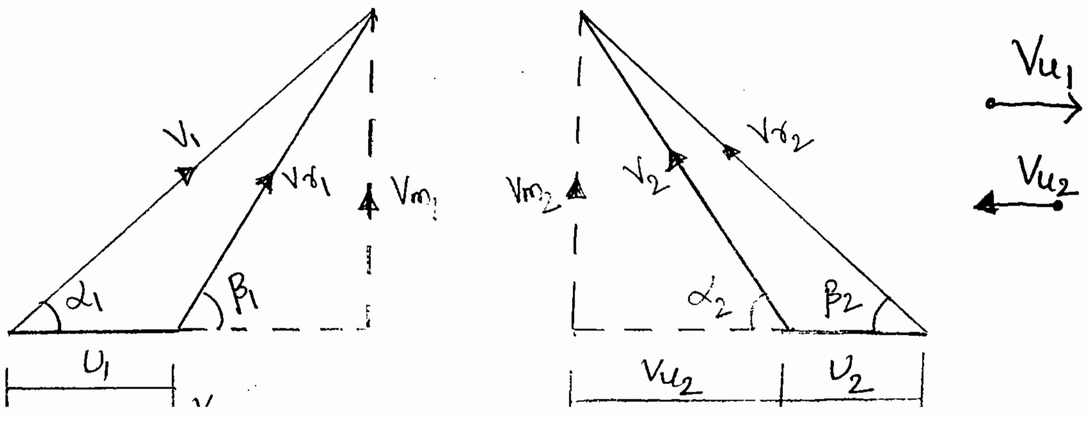


fig (a)

6. For Outward flow turbomachine

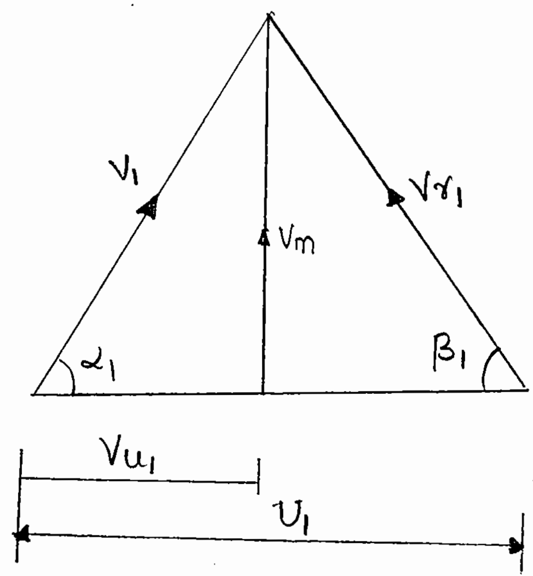
The fluid enters at inner (smaller) radius and leaves at larger (outer) radius.

ie $R_1 < R_2$, $U_1 < U_2$

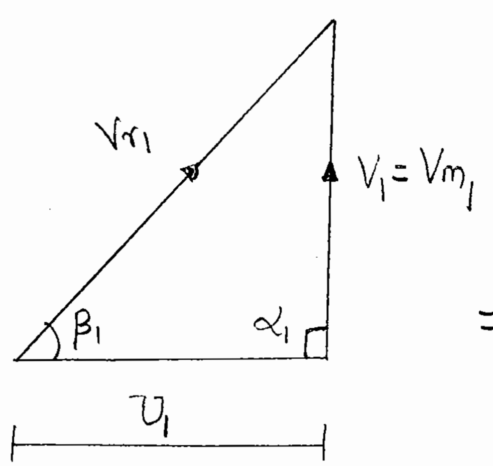


7. If $V_{u_1} > U_1$, then the velocity triangle at inlet becomes as shown in figure (a) or (b).

8. If $V_{u_1} < U_1$, then the velocity triangle at inlet becomes,



9. If the fluid enters axially/radially in case of axial/radial machines respectively, then $V_1 = V_{m_1}$ or $V_1 = V_{f_1}$ or $\alpha_1 = 90^\circ$ ie $V_{u_1} = 0$.

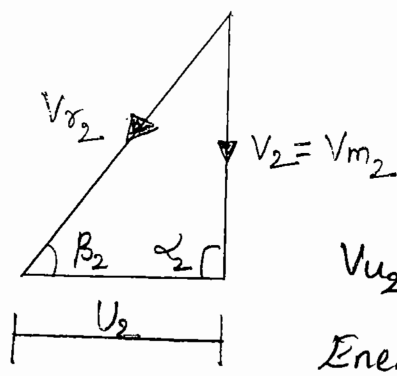


$V_{u_1} = 0$

$\Rightarrow \boxed{E = -U_2 V_{u_2}}$ power producing m/c.

10. If fluid leaves axially / radially in an axial/radial flow type machines

then $V_2 = V_{m_2}$, $\alpha_2 = 90°$ and $V_{u_2} = 0$

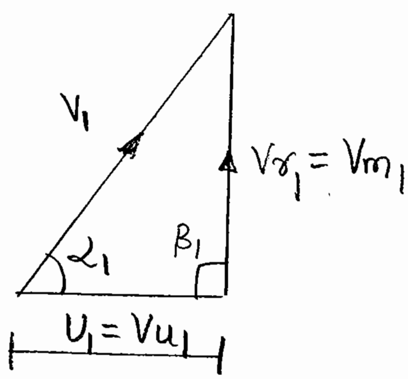


$V_{u_2} = 0$;

Energy transfer $\underline{\underline{E = U_1 V_{u_1}}}$

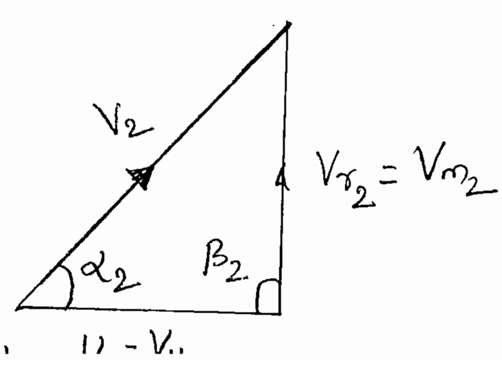
11. If the blades are radial at inlet.

ie $\beta_1 = 90°$, $V_{r_1} = V_{m_1}$



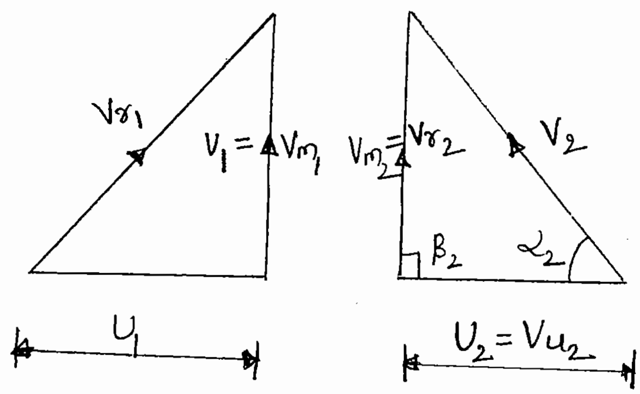
12. If the blades are radial at outlet.

ie $\beta_2 = 90°$, $V_{r_2} = V_{m_2}$



13. If fluid enters axially/radially at inlet and the blades are radial at outlet,

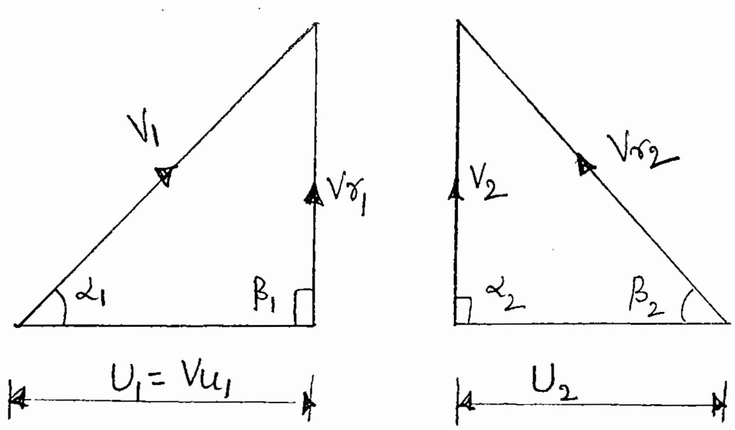
ie $\alpha_1 = 90°$, $\beta_2 = 90°$



$V_{u_1} = 0$

14. If the blades are radial at inlet and the fluid leaves rotor axially/radially.

ie $\beta_1 = 90°$, V_2 is $\perp^{er}$ or $\alpha_2 = 90°$



$V_{u_2} = 0.$

2. A tangential component of absolute velocity in the turbomachine at inlet and outlet is 28.19 m/s and 2 m/s respectively. The outer radii 0.6m and inner radii of wheel is 0.5m. The wheel rotates at 200rpm. The jet of water enters at 20° and the absolute velocity at exit is twice of its tangential component. Draw velocity triangle and calculate degree of reaction and utilization factor.

Given

$V_{u_1} = 28.19 m/s$, $N = 200 rpm$

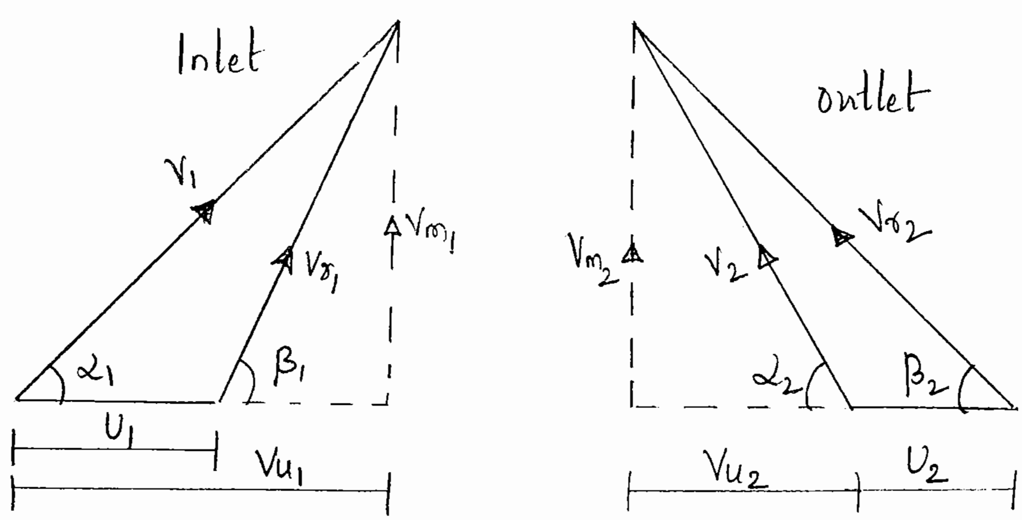
$V_{u_2} = 2 m/s$

$R_1 = 0.6m \Rightarrow U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2 \times 0.6 \times 200}{60} = 12.57 m/s$

$R_2 = 0.5m \Rightarrow U_2 = \frac{\pi \times 2 \times 0.5 \times 200}{60} = 10.47 m/s.$

$\alpha_1 = 20°$

$V_2 = 2 V_{u_2}$



from velocity triangles

i) inlet

$$\tan\alpha_1 = \frac{V_{m_1}}{V_{u_1}}$$

$$\tan 20 = \frac{V_{m_1}}{28.19}$$

$$\boxed{V_{m_1} = 10.26\ m/s}$$

$$\sin\alpha_1 = \frac{V_{m_1}}{V_1}$$

$$V_1 = \frac{10.26}{\sin 20}$$

$$\boxed{V_1 = 30\ m/s}$$

also, $V_{r_1}^2 = V_{m_1}^2 + (V_{u_1} - U_1)^2$

$$= 10.26^2 + (28.19 - 12.57)^2$$

$$\boxed{V_{r_1} = 18.69\ m/s}$$

ii) outlet

given $V_2 = 2V_{u_2}$

$$= 2 \times 2 = 4$$

$$\boxed{V_2 = 4\ m/s}$$

$$V_{m_2}^2 = V_2^2 - V_{u_2}^2$$

$$= 4^2 - 2^2$$

$$V_{m_2} = 3.464\ m/s.$$

$$V_{r_2}^2 = V_{m_2}^2 + (V_{u_2} + U_2)^2$$

$$= 12 + (2 + 10.47)^2$$

$$V_{r_2}^2 = 167.5$$

$$\boxed{V_{r_2} = 12.94\ m/s}$$

we have, Degree of reaction

$$R = \frac{(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}$$

$$= \frac{(12.57^2 - 10.47^2) + (12.94^2 - 18.69^2)}{(30^2 - 4^2) + (12.57^2 - 10.47^2) + (12.94^2 - 18.69^2)}$$

$$\boxed{R = -0.1779}$$

$\Rightarrow$

[R is negative ie it is a power generating machine]

* Since $R < 1$

hence energy transfer

$$E = \frac{U_1 V_{u_1} - U_2 V_{u_2}}{g_e}, \quad V_{u_1} \rightarrow \quad V_{u_2} \leftarrow$$

$$= \frac{(12.57 \times 28.19) + (10.47 \times 2)}{2}.$$

$$\boxed{E = 375.29\,W}$$

$\Rightarrow$ M/c is power generating, it depends on sign of E

we know

$$\epsilon = \frac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{V_1^2 + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}$$

$$= \frac{(30^2 - 4^2) + (12.57^2 - 10.47^2) + (12.94^2 - 18.69^2)}{30^2 + (12.57^2 - 10.47^2) + (12.94^2 - 18.69^2)}$$

$$\boxed{\epsilon = 0.98}$$

OR

$$\epsilon = \frac{E}{E + V_2^2/2} = \frac{375.29}{375.29 + 4^2/2} = \frac{375.29}{383.29}$$

$$\boxed{\epsilon = 0.98}$$

3. At the nozzle exists at certain stage in steam turbine, the absolute steam velocity is 300 m/s and the nozzle angle is 18°. The rotor speed is 150 m/s if the rotor blade angle is 3.5° less than inlet blade angle. Find the power output for a stage for steam mass flow is 5 kg/s. Assuming $V_{r_1} = V_{r_2}$, find 'ε'

<u>Given</u>,

Steam turbine ie $U_1 = U_2 = U$

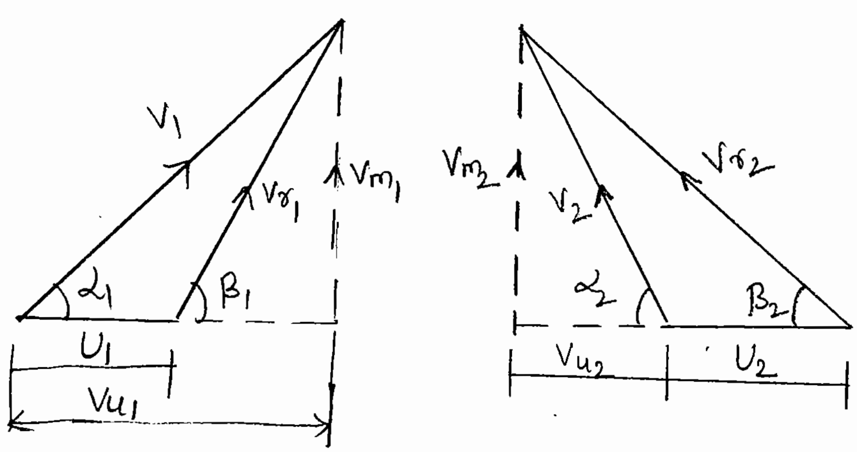
$V_1 = 300\,m/s$

$\alpha_1 = 18°$

$U = 150\,m/s$

$\beta_1 - \beta_2 = 3.5°$

To find, P, ; ε



From inlet velocity Δle.

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$\cos 18 = \frac{V_{u_1}}{300}$$

$$\boxed{V_{u_1} = 285.32\,m/s}$$

again $\sin\alpha_1 = \frac{V_{m_1}}{V_1}$

$$V_{m_1} = V_1 \sin\alpha_1 = 285.32 \times \sin 18$$

$$\boxed{V_{m_1} = 92.71\ m/s}$$

$$V_{r_1}^2 = V_{m_1}^2 + (V_{u_1} - u)^2$$

$$= 92.71^2 + (285.32 - 150)^2$$

$$\boxed{V_{r_1} = 164.03\ m/s = V_{r_2}}$$

$$\sin\beta_1 = \frac{V_{m_1}}{V_{r_1}}$$

$$= \frac{92.71}{164.03}$$

$\Rightarrow \quad \underline{\underline{\beta_1 = 34.42^\circ}} \qquad \Rightarrow \underline{\underline{\beta_2 = 30.92^\circ}}$

again from outlet vel Δle.

$$\cos\beta_2 = \frac{V_{u_2} + U_2}{V_{r_2}}$$

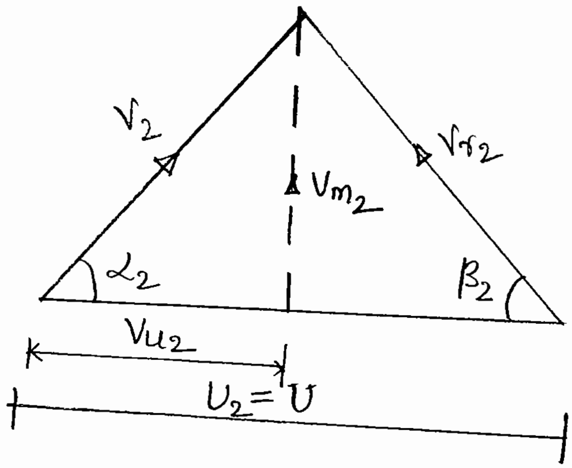
$$\cos 30.92 = \frac{V_{u_2} + 150}{164.03}$$

$$\underline{\underline{V_{u_2} = -9.28\ m/s}}$$

Negative sign indicates direction of V_{u_2} is opposite

Then we write outlet velocity triangle.

⇒



Hence,

$$\text{Power } P = \dot{m} E = \dot{m}\left[\frac{U_1 V_{u1} - U_2 V_{u2}}{g_c}\right]$$

$$= 8.5\left[285.32 - 9.28\right] \times 150$$

$$\boxed{P = 351.951\,kW}$$

We have Utilization factor $\epsilon = \dfrac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}{V_1^2 + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}$

Since $U_1 = U_2$, $V_{r1} = V_{r2}$

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2}$$

From o/l vel. triangle.

$$V_{m2}^2 = V_{r2}^2 - (U - V_{u2})^2$$

also $V_2^2 = V_{u2}^2 + V_{m2}^2$

$$V_2^2 = V_{u2}^2 + V_{r2}^2 - (U - V_{u2})^2$$

$$= 9.28^2 + 164.03^2 - (150 - 9.28)^2$$

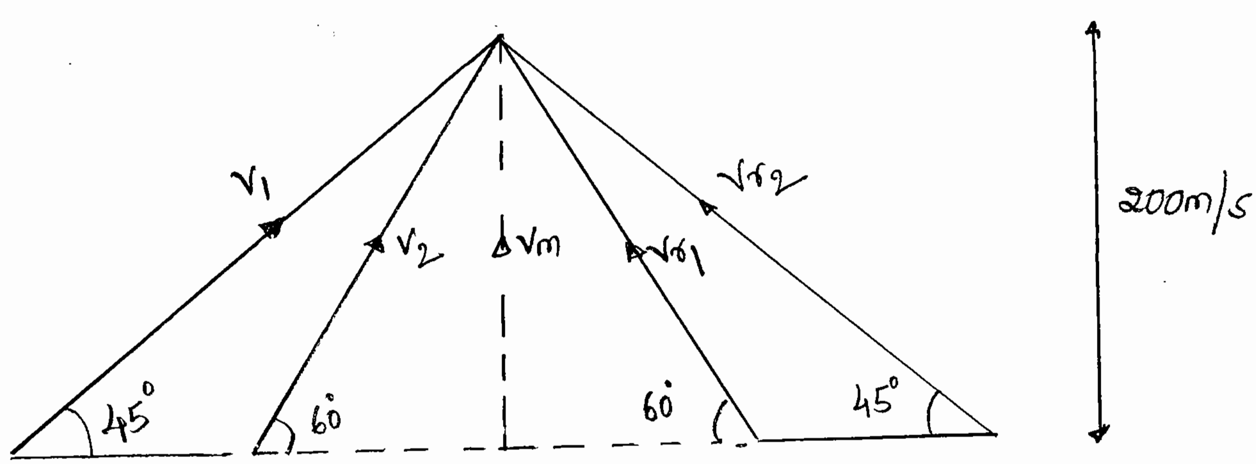
$$V_2^2 = 7189.84$$

$$\boxed{V_2 = 84.79 \text{ m/s}}$$

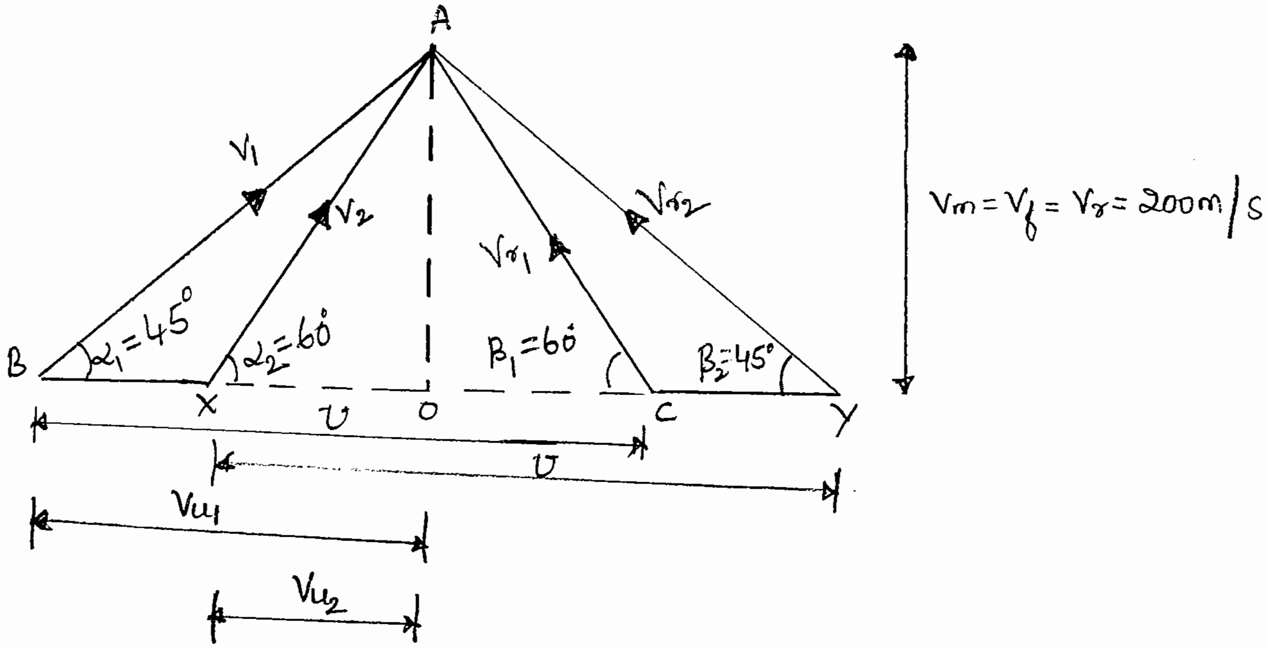
$$\epsilon = \frac{300^2 - 84.79^2}{300^2}$$

$$\boxed{\epsilon = 0.92}$$

4. A fluid flows through one stage of turbomachines, the velocity diagram is as shown (a) Is this a power generating or power absorbing turbomachine (b) what is the change in total enthalpy of the stage (c) Evaluate the degree of reaction d) Utilization factor.



Rewrite the velocity triangle.



From the velocity triangle,

ABC Δ^{le} be the inlet velocity triangle and

Δ^{le} AXY be the outlet velocity triangle.

also, $\alpha_1 = 45^\circ$, $\alpha_2 = 60^\circ$ $V_m = 200\,m/s$

$\beta_1 = 60^\circ$ $\beta_2 = 45^\circ$,

From inlet velocity Δ^{le} ΔABC

$$\tan\alpha_1 = \frac{AO}{BO} = \frac{V_m}{V_{u_1}}$$

$$\tan 45^\circ = \frac{200}{V_{u_1}}$$

$$\boxed{V_{u_1} = 200\ m/s}$$

also $\sin\alpha_1 = \frac{AO}{AB} = \frac{V_m}{V_1}$

$$\sin 45^\circ = \frac{200}{V_1}$$

$$\boxed{V_1 = 282.843\ m/s}$$

also $\tan\beta_1 = \frac{AO}{OC} = \tan\alpha_2$

since $\beta_1 = \alpha_2$, ie $OC = OX$

$\Rightarrow \quad \tan 60^\circ = \frac{V_m}{XO = OC} = \frac{200}{OC}$

$$\boxed{OC = 115.47\ m/s} = V_{u_2}$$

$$\boxed{V_{u_2} = 115.47\ m/s}$$

again from Δ^{le}, $BC = BO + OC$

but $BC = U$, $U = V_{u_1} + V_{u_2}$

$U = 200 + 115.47$

$$\boxed{U = 315.47\ m/s}$$

we have energy transfer

$E = U[V_{u_1} - V_{u_2}]$

Since $V_{u_1} \rightarrow$, $V_{u_2} \leftrightarrow$

Both have same direction

$$E = u[V_{u_1} + V_{u_2}] = 315.47(200 - 115.47)$$

$$\boxed{E = 26.667 \, kJ/kg}$$

a) Since 'E' is positive, hence it is power generating machine.

b) Change in enthalpy 'Δh' is nothing but energy transfer

ie $\Delta h = 26.667 \, kJ/kg$.

~~wk~~ we have degree of reaction,

$$R = \frac{(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}$$

again from Δ^{le} AXO

$$\sin \alpha_2 = \frac{AO}{XA} = \frac{V_m}{V_2}.$$

$$\sin 60 = \frac{200}{V_2}$$

$$\boxed{V_2 = 230.94 \, m/s}$$

$$\sin \beta_1 = \frac{V_m}{V_{r_1}} \qquad \sin \beta_2 = \frac{V_m}{V_{r_2}}$$

$$\sin 60 = \frac{200}{V_{r_1}} \qquad \sin 45 = \frac{200}{V_{r_2}}$$

$$\boxed{V_{r_1} = 230.94 \, m/s} \qquad \boxed{V_{r_2} = 282.843 \, m/s}$$

$\Rightarrow$

$$R = \frac{(315.47^2 - 315.47^2) + (282.843^2 - 230.94^2)}{(282.843^2 - 230.94^2) + (315.47^2 - 315.47^2) + (282.843^2 - 230.94^2)}$$

$$\boxed{R = 0.5}$$

Hence it is a 50% Reaction turbine

also Utilization factor $U = \frac{E}{E + V_2^2/2}$

$$= \frac{26.667 \times 10^3}{26.667 \times 10^3 + \left(\frac{230.94^2}{2}\right)}$$

$$\boxed{U = 0.5}$$

5. At a certain stage, velocity of steam outflow from nozzle in Delaval turbine is 1200 m/s and nozzle angle is 22°. If the rotor blades are equiangular and rotor tangential speed is 400 m/s. Compute

(i) Angles β_1 and β_2.

(2) Power output, assume $V_{r_1} = V_{r_2}$.

(3) The tangential force on the blade ring.

(4) Utilization factor.

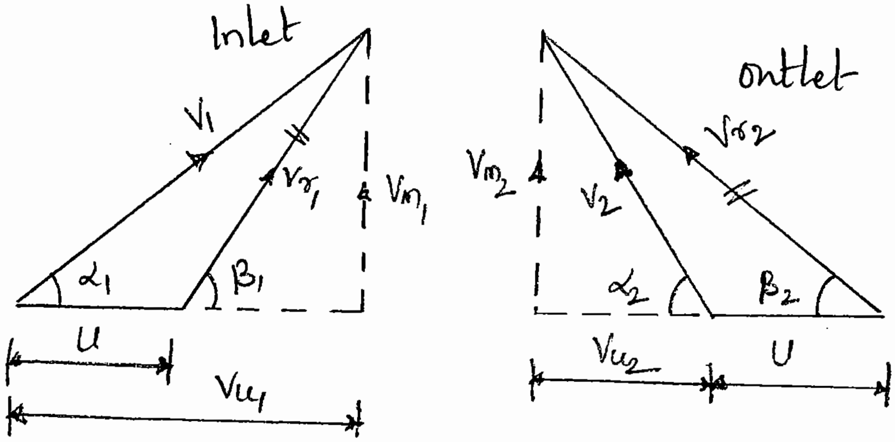
given

De laval turbine: Axial flow, impulse turbine

ie $U_1 = U_2 = U$ and $R = 0$.

$V_1 = 1200\,m/s$. $\alpha_1 = 22^\circ$, $V_{r_1} = V_{r_2}$

$U = 400\,m/s$, $\beta_1 = \beta_2$ rotor blades are equiangular.



From inlet velocity triangle,

$$\sin\alpha_1 = \frac{V_{m_1}}{V_1} \qquad \cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$V_{m_1} = V_1 \sin\alpha_1 \qquad \cos 22 = \frac{V_{u_1}}{1200}$$

$$= 1200 \sin 22^\circ$$

$$\boxed{V_{m_1} = 449.528\,m/s} \qquad \boxed{V_{u_1} = 1112.621\,m/s}$$

again $\tan\beta_1 = \dfrac{V_{m_1}}{(V_{u_1} - U)} = \dfrac{449.528}{(1112.621 - 400)}$

$$\boxed{\beta_1 = 32.24^\circ = \beta_2}$$

also $\sin\beta_1 = \frac{\;\;}{V_{r_1}}$

$$V_{r_1} = \frac{449.528}{\sin 32.24}$$

$$\boxed{V_{r_1} = 842.654 = V_{r_2}}$$

From outlet sle,

$$\cos\beta_2 = \frac{V_{u_2} + U}{V_{r_2}}.$$

$$\cos 32.24 = \frac{V_{u_2} + 400}{842.654}$$

$$\boxed{V_{u_2} = 312.734 \, m/s}$$

we have power $P = \dot{m}E = \dot{m}\,u\left[V_{u_1} \pm V_{u_2}\right]$

Since $V_{u_1} \rightarrow$

$V_{u_2} \leftarrow$

$$P = \dot{m}\,u\left[V_{u_1} + V_{u_2}\right]$$

$$= 1 \cdot 400\left[1112.621 + 312.734\right]$$

$\underline{\underline{P = 570.142 \, kW}}$ per unit mass flow rate

also $\epsilon = \frac{E}{E + \frac{V_2^2}{2}} = 0.79$

we have tangential force $F_T = \dot{m}[V_{u_1} \pm V_{u_2}]$

$$\Rightarrow = \dot{m}[V_{u_1} + V_{u_2}]$$

$$F_T = 1 [1112.621 + 312.734]$$

$$F_T = \underline{\underline{1425.355}} \text{ N per kg/sec} \quad (\text{per unit mass flow rate})$$

6. In a certain turbomachine, the fluid enters with absolute velocity having an axial component of 10 m/s and tangential component in the direction of motion is equal to 16 m/s. Tangential speed of rotor at inlet and outlet are 33 m/s and 8 m/s respectively, the absolute velocity of fluid is 16 m/s in <u>axial direction</u>. Evaluate the energy transfer between fluid and rotor. Is this power producing or power absorbing turbomachine

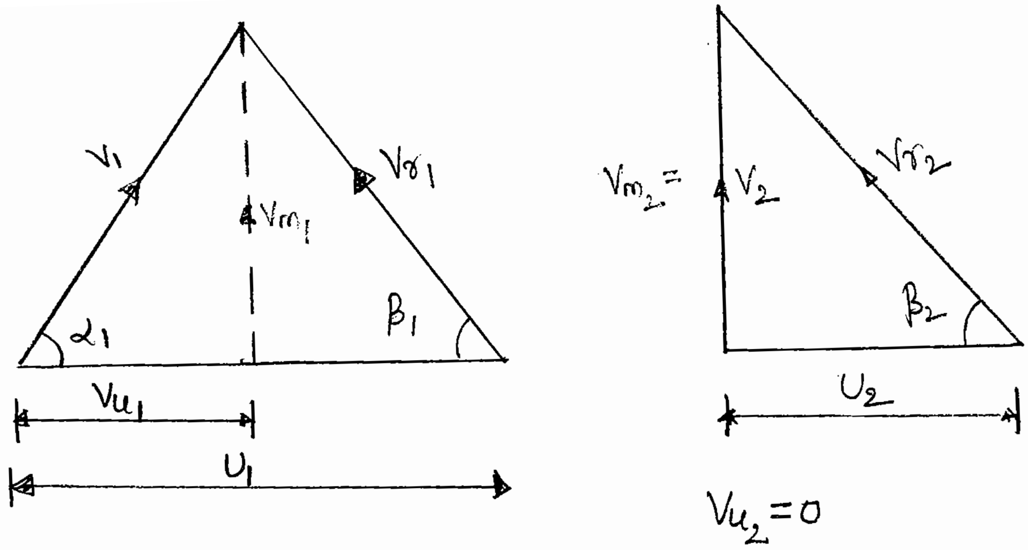
<u>Given</u>.

$V_{m_1} = 10$ m/s , $U_1 = 33$ m/s

$V_{u_1} = 16$ m/s $\quad U_2 = 8$ m/s

$V_2 = 16$ m/s in axial direction $= V_{m_2}$

Since $U_1 > V_{u_1}$



we have.

Energy transfer $E = \dfrac{[U_1 V_{u_1} \pm U_2 V_{u_2}]}{g_c}$

$E = [33 \times 16 - 8 \times 0]$

$$\boxed{E = 528 \, J/kg}$$

again we have $V_1^2 = V_{m_1}^2 + V_{u_1}^2$

$V_1^2 = 10^2 + 16^2$

$\boxed{V_1 = 18.868 \, m/s}$ and $V_2 = 16 \, m/s$ given

hence

$$\epsilon = \frac{E}{E + \left(\frac{V_2^2}{2}\right)} = \frac{528}{528 + \frac{16^2}{2}} = 0.805$$

$$\boxed{\epsilon = 0.805}$$

We know.

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - R V_2^2}.$$

$$0.805 = \frac{356 - 256}{356 - R \cdot 256}$$

$$\boxed{R = 0.905}$$

50% reaction

7. In a turbine certain stage, a tangential blade speed is 98.5 m/s. The steam velocity at the nozzle exist is 155 m/s and the nozzle angle is 18°. Assume symmetric inlet and outlet velocity triangles. Compute the inlet blade angle of rotor and power developed by stage. Assuming steam flow rate of 10 kg/sec. Find also the utilization factor.

Given.

$R = 0.5$. $u_1 = 98.5 \, m/s = u_2 = U$.

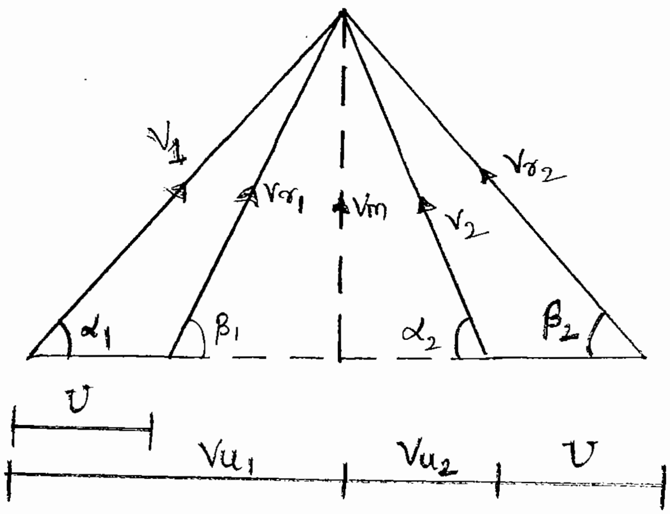
$V_1 = 155 \, m/s$, $\alpha_1 = 18°$, $\dot{m} = 10 \, kg/s$

Symmetric velocity triagles.

ie $V_1 = V_{r_2}$, $V_{r_1} = V_2$

$\alpha_1 = \beta_2$, $\alpha_2 = \beta_1$

To find β_1, P, ϵ



Hence $V_1 = V_{r_2} = 155\,m/s$, $U = 98.5\,m/s$

from inlet velocity triangle.

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$\cos 18 = \frac{V_{u_1}}{155}$$

$$\boxed{V_{u_1} = 147.414\ m/s}$$

also $$\sin\alpha_1 = \frac{V_m}{V_1}$$

$$\sin 18 = \frac{V_m}{155}$$

$$\boxed{V_m = 47.898\ m/s}$$

$$\tan\beta_1 = \frac{V_m}{V_{u_1} - u} = \frac{47.898}{147.414 - 98.5}$$

$$\boxed{\beta_1 = 44.4^\circ}$$

again $V_{r_2}^2 = V_m^2 + (V_{u_2}+U)^2$

$155^2 = 47.9^2 + (V_{u_2}+98.5)^2$.

$V_{u_2}+98.5 = 147.413$

$$\boxed{V_{u_2} = 48.913\ m/s}$$

we know, $P = \dot{m}E = \dot{m}u[V_{u_1}+V_{u_2}]$ $V_{u_1} \rightarrow$

$P = 10\times 98.5[147.414+48.913]$ $V_{u_2} \leftarrow$

$$\boxed{P = 193.382\ kW}$$

also $E = u[V_{u_1}+V_{u_2}] = \underline{\underline{19.3382}}\ kJ/kg$

Utilization factor $\epsilon = \dfrac{E}{E+V_2^2/2}$ or $\dfrac{V_1^2-V_2^2}{V_1^2-RV_2^2}$.

also from Δ^{le}, $\sin\alpha_2 = \dfrac{V_m}{V_2}$.

$\sin 44.4 = \dfrac{47.898}{V_2}$

$$\boxed{V_2 = 68.459\ m/s}$$

$\Rightarrow$

$\epsilon = \dfrac{19.3382\times10^3}{19.3382\times10^3+\dfrac{68.459^2}{2}}$ OR $\epsilon = \dfrac{155^2-68.459^2}{155^2-(0.5\times68.459^2)}$

$$\boxed{\epsilon = 0.892}$$

8. Air enters a rotor in an axial flow turbine with a tangential velocity of turbine is equal to 600 m/s in the direction of rotation. At the rotor exit the tangential component of absolute velocity is 100 m/s in the direction opposite to that of rotation. The tangential blade speed is 250 m/s. Evaluate

a) Change in total enthalpy of air expressed in kcal/kg between inlet and exit of the rotor.

b) Change in total temperature across the rotor.

c) Power in kW if the flow rate is 10 kg/s.

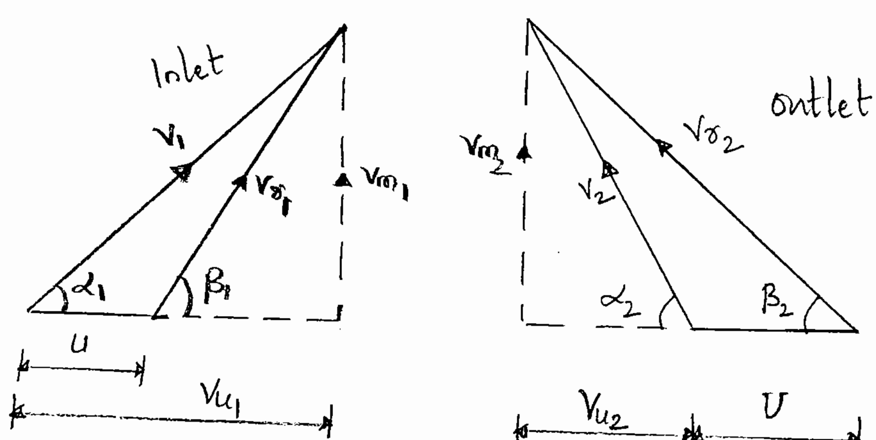
Given

Axial flow, $U_1 = U_2 = U$, fluid : air

$V_{u_1} = 600\,m/s \longrightarrow$ rotor direction.

$V_{u_2} = 100\,m/s \longleftarrow$ rotor motion dir^n.

$U = 250\,m/s \rightarrow$



Change in enthalpy is equal to energy transfer between rotor and fluid.

ie $\Delta h = E = U[V_{u_1} \pm V_{u_2}]$

$E = 250[600 + 100]$ $\quad\because$ Both are in same dir^n.

$\underline{\underline{E = 175\ kJ/kg}}$.

or $E = 41.808\ kcal/kg$

or $\Delta h = -41.808\ kcal/kg$

-ve sign indicates that enthalpy ~~...~~ is decreased.

also we know, $\Delta T = ?$,

$\Delta h = C_p\, \Delta T$

$-175 \times 10^3 = 1.005 \times 10^3 \times \Delta T$

$\Delta T = -174.13K$ (Temperature decreases)

we have Power $P = \dot{m} E$

$= 10 \times 175 \times 10^3$

$\boxed{P = 1750\ kW}$

9. In a radial inward flow francis turbine, the runner outward diameter is 75 cm and inner diameter is 50 cm. The runner speed is 400 rpm. Liquid water enters the wheel

at a speed of 15 m/s at an angle of 15° to wheel tangent at point of entry. The discharge at the outlet is radial and absolute velocity is 5 m/s. Find the runner blade angles at the inlet and draw the velocity triangle. What is the power output per unit mass flow rate of water through the turbine also find R and ϵ.

<u>Given</u>

Radial inward flow Francis turbine.

$D_1 = 75cm = 0.75m$, $N = 400 rpm$

$D_2 = 50cm = 0.5m$ $\qquad V_1 = 15 m/s$

$\alpha_1 = 15°$, $\qquad V_2 = 5 m/s$

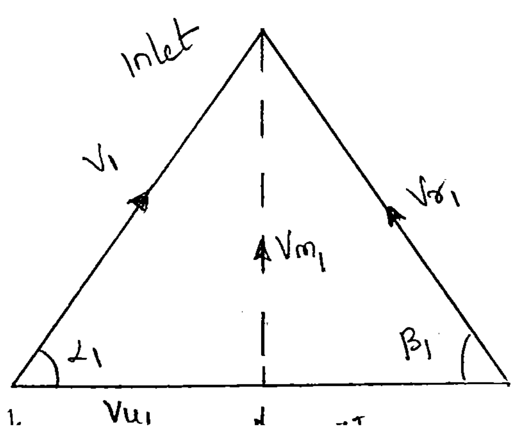
Discharge at outlet is radial $\Rightarrow$ V_2 is $\perp^{er}$ ie $\alpha_2 = 90°$

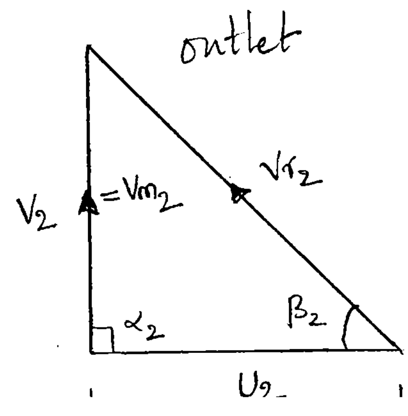
To find: β_1, $\frac{P}{m}$, R, ϵ.

we have $\quad U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.75 \times 400}{60} = 15.708 m/s$

$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 60}{60} = 10.472 m/s$

[Since $U_1 > V_{u_1}$, therefore vel. Δle becomes : After Calc. U_1, V_{u_1}]





we have from inlet velocity triangle

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$\cos 15 = \frac{V_{u_1}}{15} \qquad \boxed{V_{u_1} = 14.489\ m/s}$$

we know power $p = \dot{m}E = \dot{m}[u_1 V_{u_1} \pm u_2 V_{u_2}]$

$$p = u_1 V_{u_1} \qquad \text{since } V_{u_2} = 0,\ \dot{m} = 1\ kg/s$$

$$p = 15.708 \times 14.489$$

$$\boxed{p = 227.593\ W/kg} \quad \text{or W per kg/sec}$$

also

$$\sin\alpha_1 = \frac{V_m}{V_1}$$

$$V_m = 15 \sin 15$$

$$\boxed{V_m = 3.882\ m/s}$$

$$\tan\beta_1 = \frac{V_m}{u_1 - V_{u_1}} = \frac{3.882}{15.708 - 14.489}$$

$$\boxed{\beta_1 = 72.57^\circ}$$

also $E = u_1 V_{u_1} = 227.593\ J/kg.$

again $\epsilon = \dfrac{\cancel{V_1}\ E}{E + \frac{V_2^2}{2}}$

$$\overline{227.543 + \frac{5^2}{2}}$$

$$\boxed{\epsilon = 0.948}$$

also

R and ϵ relation, $\epsilon = \dfrac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$.

$$0.948 = \frac{15^2 - 5^2}{15^2 - R5^2}.$$

$$\boxed{R = 0.561}$$

10. At a stage of impulse turbine, the mean blade diameter 80cm, its rotational speed is 50rps. Absolute velocity of fluid from nozzle inclined at 20° to the plane of wheel is 300 m/s. If the utilization factor is 0.85 and relative velocity at rotor exit equal that at inlet. Find inlet and exit rotor angles, also find power output for the mass flow rate of 1 kg/s.

<u>Given</u>:

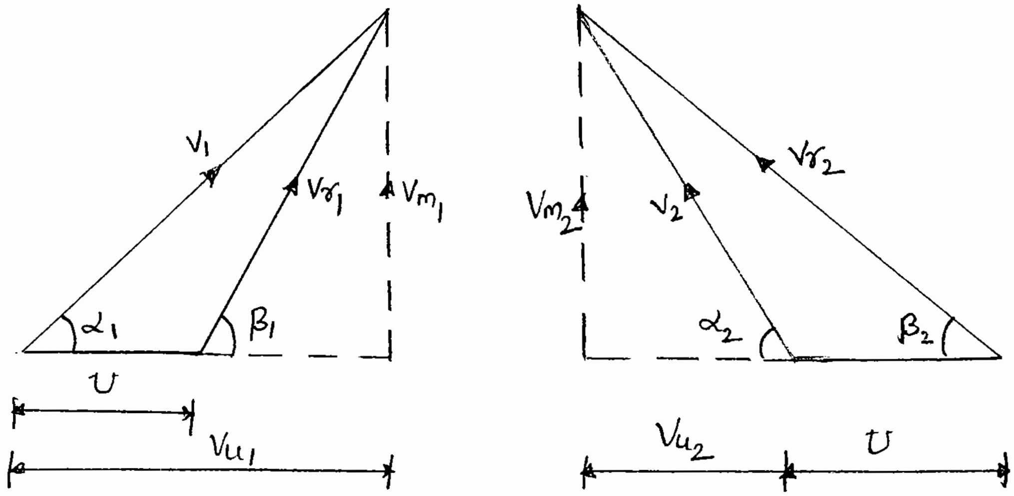
Impulse turbine ie Degree of Reaction $R = 0$

$D_{mean} = 80\,cm = 0.8\,m$, ie $U_1 = U_2 = U$.

$N = 50\,rps$

$V_1 = 300\,m/s$, $\alpha_1 = 20°$

$\epsilon = 0.85$, $V_{r_1} = V_{r_2}$



we know, $U = \dfrac{\pi D N}{60} = \pi \times 0.8 \times 50$ $\qquad N \rightarrow$ in rps

$$\boxed{U = 125.664 \text{ m/s}}$$

From inlet velocity triangle,

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$V_{u_1} = 300 \times \cos 20^\circ$$

$$\boxed{V_{u_1} = 281.908 \text{ m/s}}$$

$$\sin\alpha_1 = \frac{V_{m_1}}{V_1}$$

$$\sin 20 = \frac{V_{m_1}}{300}$$

$$\boxed{V_{m_1} = 102.606 \text{ m/s}}$$

also $\quad V_{r_1}^2 = V_{m_1}^2 + (V_{u_1} - U)^2$, (from pythagorous theorem)

$$V_{r_1}^2 = 102.606^2 + (281.908 - 125.664)^2$$

$$\boxed{V_{r_1} = 186.923 \text{ m/s} = V_{r_2}}$$ given,

also $\sin\beta_1 = \frac{\ldots}{V_{r_1}}$

$$\sin\beta_1 = \frac{102.606}{186.923}$$

$$\boxed{\beta_1 = 33.29^\circ}$$

we have utilization factor

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - R V_2^2} \qquad \text{here } R=0$$

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2} = 1 - \left(\frac{V_2}{V_1}\right)^2.$$

$$\Rightarrow \quad 0.85 = 1 - \left(\frac{V_2}{V_1}\right)^2.$$

$$\left(\frac{V_2}{V_1}\right)^2 = 0.15$$

$$V_2 = \sqrt{0.15}\, V_1 = \sqrt{0.15} \times 300$$

$$\boxed{V_2 = 116.19 \text{ m/s}}$$

also

$$\epsilon = \frac{E}{E + \left(\frac{V_2^2}{2}\right)}$$

$$0.85 = \frac{E}{E + \left(\frac{116.19^2}{2}\right)}$$

$$\Rightarrow \quad E = 0.85E + 5737.549$$

$$\boxed{E = 38250.329 \text{ J/kg}}$$

we know, $E = \dfrac{UV_{u_1} + UV_{u_2}}{g_c}$ $V_{u_1} \rightarrow$

$V_{u_2} \leftarrow$

$$38250.329 = 125.664\left[281.908 + V_{u_2}\right]$$

$$\boxed{V_{u_2} = 22.478 \text{ m/s}}$$

From outlet velocity triangle,

$$\cos\beta_2 = \frac{V_{u_2} + U}{V_{r_2}}$$

$$= \frac{22.478 + 125.664}{186.923}$$

$$\boxed{\beta_2 = 37.58^\circ}$$

we know power, $P = \dot{m} E$

$$= 1 \times 38250.329$$

$$\boxed{P = 38.25 \text{ kW}}$$

11. The following data refers to a mixed flow turbomachine were fluid absolute velocity at inlet is axial while at outlet the relative velocity is radial. Inlet hub diameter 8cm, and impeller tip diameter is 25cm, Speed of the rotor is 3000, also axial velocity at inlet equal to radial velocity at exit. Find R and energy transfer if the relative velocity at exit is equal to inlet tangential blade

Given

Mixed flow turbomachine

Absolute velocity at inlet is axial ie V_1 is $\perp^{er}$.

Relative velocity at outlet is radial ie V_{r_2} is $\perp^{er}$.

inlet hub dia $D_1 = 8cm = 0.08\,m$

Impeller tip dia $D_2 = 25cm = 0.25m$

$V_{m_1} = V_{m_2} = V_m$

$V_{r_2} = U_1$, $N = 3000\,rpm$

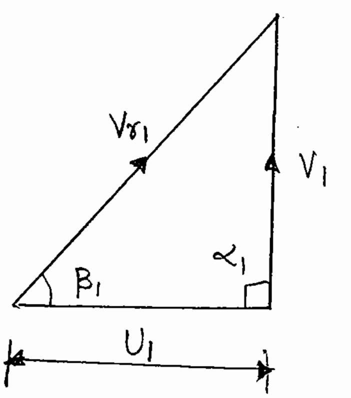
we know, $U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.08 \times 3000}{60}$

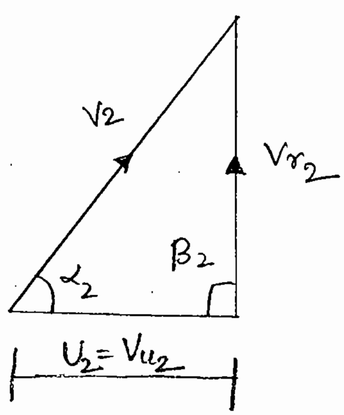
$$\boxed{U_1 = 12.5664\ m/s}$$

also

$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 3000}{60}$

$$\boxed{U_2 = 39.27\ m/s}$$





$V_{u_1} = 0$

From given conditions,

$U_1 = V_{r_2} = V_1 = V_m$, $U_2 = V_{u_2}$, $V_{u_1} = 0$

Hence Energy transfer $E = \dfrac{V_1 u_1 - V_2 u_2}{g_c}$

$$E = \frac{0 - (39.27)(39.27)}{1}$$

$\boxed{E = -1542.126\ J/kg}$ Power absorbing m/c.

also, Degree of Reaction

$$R = \frac{(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{(V_1^2 - V_2^2) + (V_{r_2}^2 - V_{r_1}^2) + (U_1^2 - U_2^2)}$$

since $U_1 = V_{r_2} = V_1$,

From inlet vel ∆le

$$V_{r_1}^2 = V_1^2 + U_1^2$$

$$= 12.5664^2 + 12.5664^2$$

$$V_{r_1} = 17.7716\ m/s$$

$|||^{ly}$ $V_2 = 41.2316\ m/s$.

$$R = \frac{(12.5664^2 - 39.27^2) + (12.5664^2 - 17.7716^2)}{(12.5664^2 - 41.23^2) + (12.5664^2 - 17.7716^2) + (12.5664^2 - 39.27^2)}$$

$$R = \frac{-1542.1338}{-3084.2643}$$

$\boxed{R = 0.5}$

12. The following data refers to a 50% degree of reaction axial flow turbomachine.

Inlet fluid velocity 230 m/s

Outlet angle of inlet guide blade is 30°

Inlet rotor angle is 60° and outlet rotor angle is 25°

Find the utilization factor, axial thrust and power output per unit mass flow rate, if $V_1 = V_{r_2}$.

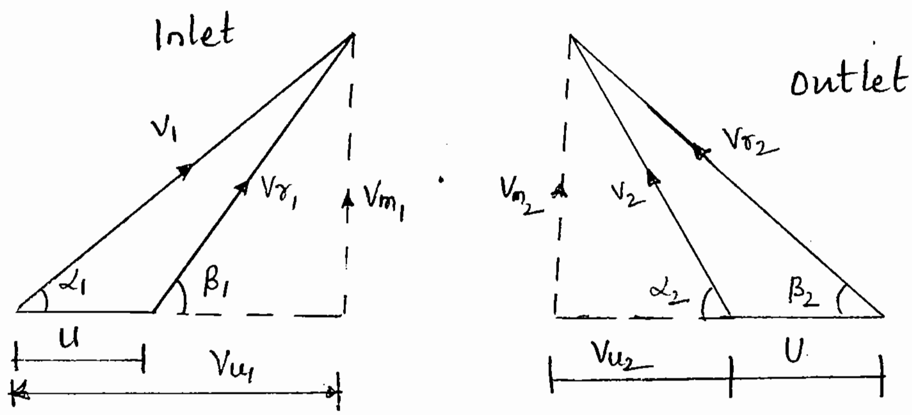
<u>Given.</u>

$R = 0.5$, Axial flow type, $V_1 = V_{r_2}$, $V_2 = V_{r_1}$

$U_1 = U_2 = U$, also $\alpha_1 = \beta_2$, $\beta_1 = \alpha_2$

$V_1 = 230\,m/s$, $\alpha_1 = 30^\circ$

$\beta_1 = 60^\circ$, $\beta_2 = 25^\circ$, To find ϵ, Fa, P



From inlet velocity triangle

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$\cos 30 = \frac{V_{u_1}}{230}$$

$$\boxed{V_{u_1} = 199.186\,m/s}$$

also $$\sin\alpha_1 = \frac{V_{m_1}}{V_1}$$

$$V_{m_1} = V_1 \sin\alpha_1 = 230 \times \sin 30$$

$$\boxed{V_{m_1} = 115\,m/s}$$

also $$\tan\beta_1 = \frac{V_{m_1}}{V_{u_1} - U}$$

$$\tan 60 = \frac{115}{199.186 - U}$$

$$\boxed{U = 132.79 \text{ m/s}}$$

From outlet velocity triangle.

$$\cos\beta_2 = \frac{V_{u_2} + U}{V_{r_2}} = \frac{V_{u_2} + U}{V_1}$$

$$\cos 25 = \frac{V_{u_2} + 132.79}{230}$$

$$\cancel{\boxed{V_{u_2} = 66.396 \text{ m/s}}}$$

also $\boxed{V_{u_2} = 75.661 \text{ m/s}}$

hence power $P = U[V_{u_1} + V_{u_2}]$

$$= 132.79[199.186 + 75.661]$$

$$\boxed{P = 36.497 \text{ kW}}$$

also from outlet vel $\triangle$le

$$V_{m_2}^2 = V_{r_2}^2 - (V_{u_2} + U)^2$$

$$= 230^2 - (75.661 + 132.79)^2$$

$$V_{m_2} = 97.202 \text{ m/s.}$$

hence axial thrust

$$F_a = \dot{m}(V_{m_1} - V_{m_2})$$

$$= 1(115 - 97.202)$$

$$\boxed{F_a = 17.8 \text{ N per kg/sec}}$$

also utilization factor

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

$$\epsilon = \frac{230^2 - 123.18^2}{230^2 - 0.5 \times 123.18^2}$$

$$\boxed{\epsilon = 0.8325}$$

13. In an axial flow turbine The discharge blade angles are 20° each for both stator and rotor. The steam speed at the exit of the fixed blade is 140 m/s, the ratio of meroedonial component of absolute velocity to the blade speed is 0.7 and 0.76 at the entry and exit of the rotor respectively. Find the inlet blade angle, power developed for a mass flow rate of 2.6 kg/sec also Degree of reaction.

<u>Given.</u>

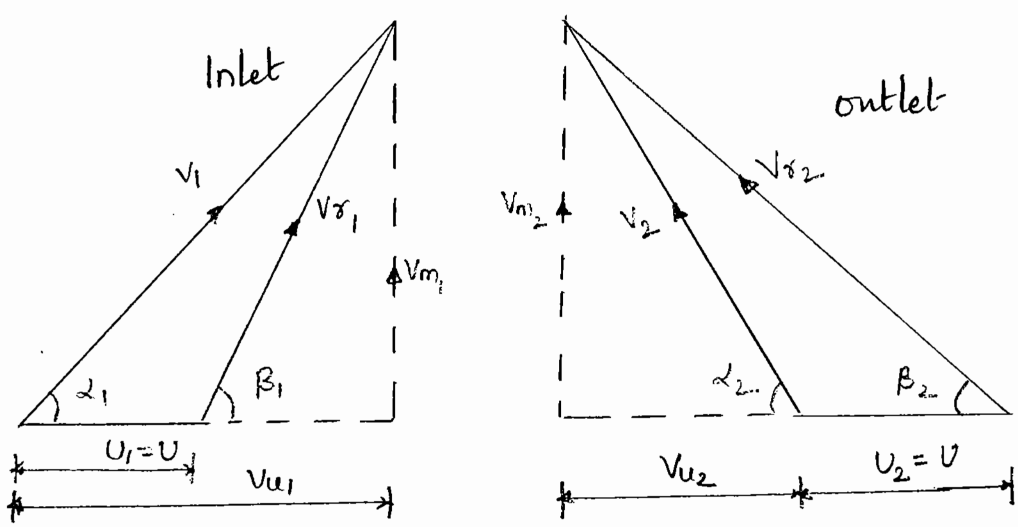
Axial flow turbine $U_1 = U_2 = U$

Discharge blade angle for stator $\alpha_1 = 20°$, $\beta_2 = 20°$

$V_1 = 140\,m/s$ (Exit of fixed blade ie nozzle)

$\frac{V_{m_1}}{U} = 0.7$, $\frac{V_{m_2}}{U} = 0.76$

To find. β_1, $P, \dot{m} = 2.6\,kg/sec$, R



From inlet velocity triangle.

$$\sin\alpha_1 = \frac{V_{m_1}}{V_1}$$

$$\sin\alpha_1 = \frac{\quad}{140}$$

$$\boxed{V_{m_1} = 47.88\ m/s}$$

also $\dfrac{V_{m_1}}{U} = 0.7$

$$\boxed{U = 68.4\ m/s}$$

again $\dfrac{V_{m_2}}{U} = 0.76$

$$\boxed{V_{m_2} = 51.99\ m/s}$$

From inlet vel Δ^{le}.

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$\cos 20 = \frac{V_{u_1}}{140}$$

$$\boxed{V_{u_1} = 131.56\ m/s}$$

again

$$\tan\beta_1 = \frac{V_{m_1}}{V_{u_1} - U}$$

$$= \frac{47.88}{131.56 - 68.4}$$

$$\boxed{\beta_1 = 37.16^\circ}$$

again from outlet velocity triangle

$$\tan\beta_2 = \frac{V_{m_2}}{V_{u_2} + U}$$

$$\tan 20^\circ = \frac{51.99}{V_{u_2} + 68.4} \qquad \boxed{V_{u_2} = 74.44\ m/s}$$

we know

$$E = \frac{[U_1 V_{u_1} \pm V_{u_2} U_2]}{g_e}.$$

$$E = U[V_{u_1} + V_{u_2}]$$ opposite direction.

$$= 68.4[131.56 + 74.44]$$

$$E = 14090.4 \text{ J/kg}$$

also

Power $P = \dot{m}E = 2.6 \times 14090.4$

$$\boxed{P = 36.63504 \text{ kW}}$$

From outlet velocity triangle.

$$V_2^2 = V_{u_2}^2 + V_{m_2}^2$$

$$= 74.44^2 + 51.99^2$$

$$\boxed{V_2 = 90.798 \text{ m/s}}$$

again $$V_{r_2}^2 = V_{m_2}^2 + (V_{u_2} + U)^2$$

$$= 51.99^2 + (74.44 + 68.4)^2$$

$$\boxed{V_{r_2} = 152.007 \text{ m/s}}$$

From Inlet vel. Δle

$$V_{r_1}^2 = V_{m_1}^2 + (V_{u_1} - U)^2$$

$$= 47.88^2 + (131.56 - 68.4)^2$$

$$\boxed{V_{r_1} = 79.257 \text{ m/s}}$$

We have degree of reaction

$$R = \frac{(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}$$

$$R = \frac{(152.007^2 - 79.257^2)}{(140^2 - 90.798^2) + (152.007^2 - 79.257^2)}$$

$$\boxed{R = 0.597}$$

$\boxed{\text{OR}}$ Find ϵ using $\epsilon = \frac{E}{E + V_2^2/2}$

$$\Rightarrow \quad \epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

$$\Rightarrow R = \underline{\underline{0.597}}$$

14. A hydraulic reaction turbine of radial inward type works on head of 160m. At the point of entry the rotor blade angle is 119°. The diameter of the runner at inlet and outlet are 3.65m and 2.45m respectively. If the absolute velocity at the wheel outlet is radial directed with a magnitude of 15.5m/s and radial component of velocity at the inlet is 10.3m/s. Find the power developed by machine assuming that 88%

of the available head of machine is converted into power and flow rate is $110 m^3/s$. Find also the R and E.

Given

Hydraulic reaction turbine

Fluid: water.

$H = 160 m$, $\beta_1' = 119°$

$D_1 = 3.65 m$ $Q = 110 m^3/s$

$D_2 = 2.45 m$ $\eta = 88\%$

absolute velocity of fluid at outlet is radial ie V_2 is $\perp^{er}$.

also $V_2 = 15.5 m/s$ ie $\alpha_2 = 90°$

$V_{m_1} = 10.3 m/s$.

To find P, R, E

we know

$$\eta = \frac{\text{Energy transfer (power) in rotor}}{\text{water energy in (fluid) water}}$$

$$\eta = \frac{P}{\rho g Q H}$$

but $\rho Q = \text{mass flow} = \dot{m}$

$$\eta = \frac{P}{\dot{m}} \cdot \frac{1}{gH} = \frac{E}{gH} \qquad \therefore \frac{P}{\dot{m}} = E$$

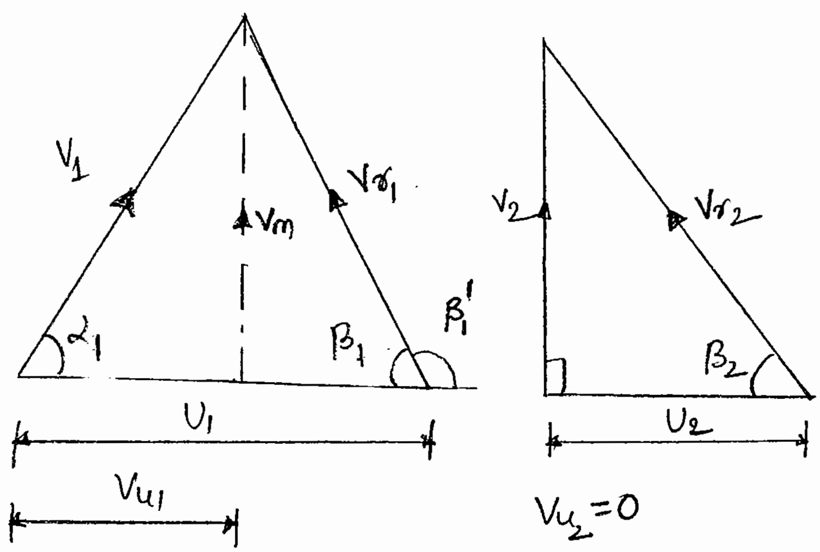
$$\boxed{\eta = \frac{E}{gH}}$$

$$0.88 = \frac{E}{9.81 \times 160}$$

$$\boxed{E = 1381.248\ J/kg}$$

also $E = U_1 V_{u_1} = 1381.248\ J/kg$ since $V_{u_2} = 0$

or $U_1 = \dfrac{1381.248}{V_{u_1}}$ — (1).



from inlet vel. Δ^{le}

$$\tan\beta_1 = \frac{V_{m_1}}{U_1 - V_{u_1}}$$

$$\tan 61 = \frac{10.3}{U_1 - V_{u_1}}$$

$$U_1 - V_{u_1} = 5.7094 \quad \text{— (2)}$$

put (1) in (2)

$$\frac{1381.248}{V_{u_1}} - V_{u_1} = 5.7094$$

$V_{u_1} + 5.7094 - 1381.248 = 0$

$$\boxed{V_{u_1} = 34.42\ m/s}$$

and hence $U_1 = \dfrac{1381.248}{V_{u_1}} = 40.12923$

$$\boxed{U_1 = 40.1292\ m/s}$$

hence power $P = \dot{m}\left[U_1 V_{u_1} \pm U_2 V_{u_2}\right]$

$= \rho Q\, U_1 V_{u_1}$

$= \cancel{1000} \times 110 \times 40.1292 \times 34.42$

$P = \underline{\underline{151.9372\ MW}}$

again we have $U_1 = \dfrac{\pi D_1 N}{60} \Rightarrow 40.1292 = \dfrac{\pi \times 3.65 \times N}{60}$

$$\boxed{N = 209.98\ RPM}$$

hence $U_2 = \dfrac{\pi D_2 N}{60} = \underline{\underline{26.936}}\ m/s$

From inlet velocity triangle.

$V_1^2 = V_{m_1}^2 + V_{u_1}^2$

$= 10.3^2 + 34.42^2$

$$\boxed{V_1 = 35.928\ m/s}$$

we know $\epsilon = \dfrac{\quad}{E + V_2^2/2}$

$$= \frac{1381.248}{1381.248 + \frac{15.5^2}{2}}$$

$$\boxed{\epsilon = 0.92}$$

again

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_1^2}.$$

$$0.92 = \frac{35.928^2 - 15.5^2}{35.928^2 - R\,15.5^2}$$

$$\boxed{R = 0.62}$$

15. A mixed flow turbine handling water operates on static head of 65 m. In steady flow the static pressure at the rotor inlet is 3.5 atm. The absolute velocity at the rotor inlet is directed at an angle of 25° to the wheel tangent so that V_{u1} is positive. The absolute velocity at the rotor exit is purely axial. If the R is 0.47 and $\epsilon = 0.896$. Compute the tangential blade speed at the inlet as well as the inlet blade ~~spe~~ angle. Find workput per unit mass flow rate of water.

Given

Mixed flow turbine handling water

$H = 65m$ static head

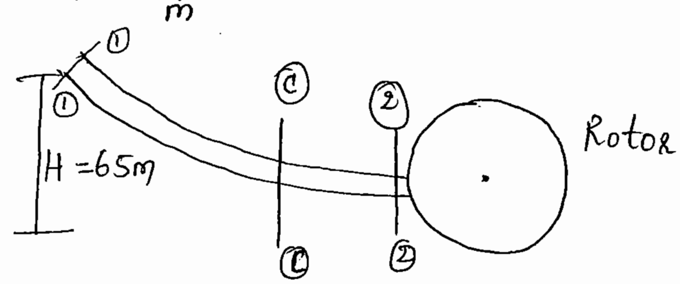
Static pressure $P_0 = 3.5\,atm = 3.5\,bar$

$\alpha_1 = 25^\circ$, $V_{u_1} = +ve$

V_2 is axial ie perpendicular ie $\alpha_2 = 90^\circ$

$R = 0.47$, $\epsilon = 0.896$

To find V_1, β_1, $\frac{W}{\dot{m}}$,



Applying Bernoullis equation

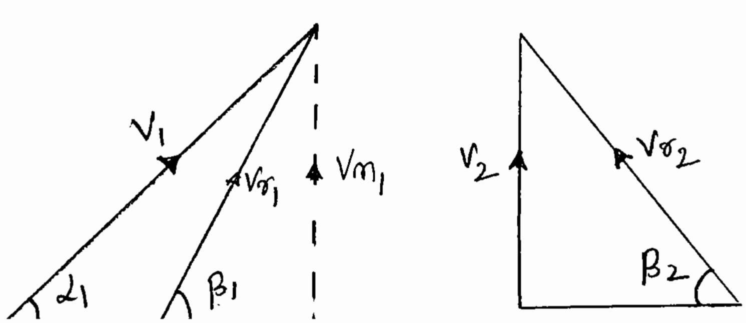
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + H_2 .$$

$$65 = \frac{3.5 \times 10^5}{1000 \times 9.81} + \frac{V_2^2}{2 \times 9.81} + 0$$

$$\boxed{V_2 = 23.985\,m/s}$$

outlet of the nozzle is inlet to rotor ie

$$\boxed{V_1 = 23.985\,m/s}$$



we have

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

$$0.896 = \frac{23.985^2 - V_2^2}{23.985^2 - 0.47V_2^2}$$

$$515.451 - 0.42112V_2^2 = 575.28 - V_2^2$$

$$0.5788V_2^2 = 59.829$$

$$\boxed{V_2 = 10.17 m/s}$$

again $$\epsilon = \frac{E}{E + \frac{V_2^2}{2}}$$

$$0.896 = \frac{E}{E + \frac{10.17^2}{2}}$$

$$\Rightarrow \quad 0.896E + 46.336 = E$$

$$\boxed{E = 445.54 J/kg}$$

work output per unit mass flow rate $= \frac{W}{\dot{m}} = E$

$$\boxed{W = 445.54 W}$$

From Inlet velocity triangle.

$$\cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

$$\cos 25^\circ = \frac{V_{u_1}}{23.985}$$

$$\boxed{V_{u_1} = 21.738 \text{ m/s}}$$

also $\quad E = U_1 V_{u_1} + U_2 V_{u_2} \quad$ but $V_{u_2} = 0$

$$E = U_1 V_{u_1}$$

$$445.54 = U_1 \cdot 21.738$$

$$\boxed{U_1 = 20.5 \text{ m/s}}$$

From inlet velocity triangle.

$$\tan\beta_1 = \frac{V_{m_1}}{V_{u_1} - U_1}$$

also $\quad V_{m_1}^2 = V_1^2 - V_{u_1}^2$

$$= 23.985^2 - 21.738^2$$

$$\boxed{V_{m_1} = 10.14 \text{ m/s}}$$

$$\tan\beta_1 = \frac{10.14}{21.738 - 20.5}$$

$$\boxed{\beta_1 = 83.04^\circ}$$

16. In an slow speed inward flow radial hydraulic turbine, degree of reaction is R and utilization factor ε. Assuming the radial velocity component is constant throughout and there is

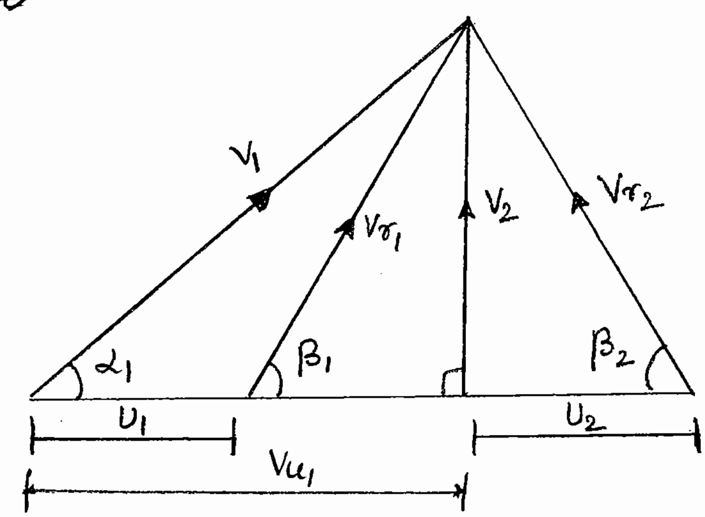
no tangential component of absolute velocity at the outlet. Show that the inlet nozzle angle is given by

$$\alpha_1 = \cot^{-1}\sqrt{\left(\frac{1-R}{1-\epsilon}\right)\epsilon}$$

Since there no whirl velocity at outlet, it refers to the maximization condition.

Hence we can combine vel. △les.

$\Rightarrow$



General utilization factor is given by

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - R V_2^2} \quad \text{— ①}$$

From velocity triangle, $\sin\alpha_1 = \frac{V_2}{V_1}$

$$V_2 = V_1 \sin\alpha_1 \quad \text{— ⓐ}$$

put ⓐ in ① $\Rightarrow$ $\epsilon = \dfrac{V_1^2 - V_1^2 \sin^2\alpha_1}{V_1^2 - R V_1^2 \sin^2\alpha_1}$

$$\epsilon = \frac{1-\sin^2\alpha_1}{1-R\sin^2\alpha_1} = \frac{\cos^2\alpha_1}{1-R\sin^2\alpha_1}$$

$$\frac{1}{\epsilon} = \frac{1-R\sin^2\alpha_1}{\cos^2\alpha_1}$$

$$\frac{1}{\epsilon} = \frac{(1-R\sin^2\alpha_1)\,\sin^2\alpha_1}{(\cos^2\alpha_1/\sin^2\alpha_1)}$$

$$\frac{1}{\epsilon} = \frac{\frac{1}{\sin^2\alpha_1} - R}{\cot^2\alpha_1}$$

$$\frac{\cot^2\alpha_1}{\epsilon} = \operatorname{cosec}^2\alpha_1 - R$$

$$= 1+\cot^2\alpha_1 - R$$

$$\frac{\cot^2\alpha_1}{\epsilon} = (1-R) + \cot^2\alpha_1$$

$$\frac{1}{\epsilon} = \frac{1-R}{\cot^2\alpha_1} + 1$$

$$\frac{1-\epsilon}{\epsilon} = \frac{1-R}{\cot^2\alpha_1}$$

$$\cot^2\alpha_1 = \frac{1-R}{1-\epsilon}\cdot\epsilon$$

$$\boxed{\alpha_1 = \sqrt{\left(\frac{1-R}{1-\epsilon}\right)\epsilon}}$$

11. A Radial outward flow turbomachine has no whirl velocity at inlet (V_{u_1}) and the blade speed at the exit is twice as that of inlet. The radial velocity component of absolute velocity is constant throughout. At the rotor exit the tangential velocity component is same direction as rotor rotation. Taking the inlet blade angle as 45°. Show that

$$E = -2U_1^2 (2 - \cot\beta_2) \quad \text{and}$$

$$R = \frac{\cot\beta_2 + 2}{4}$$ also discuss effect of β_2 on E, R

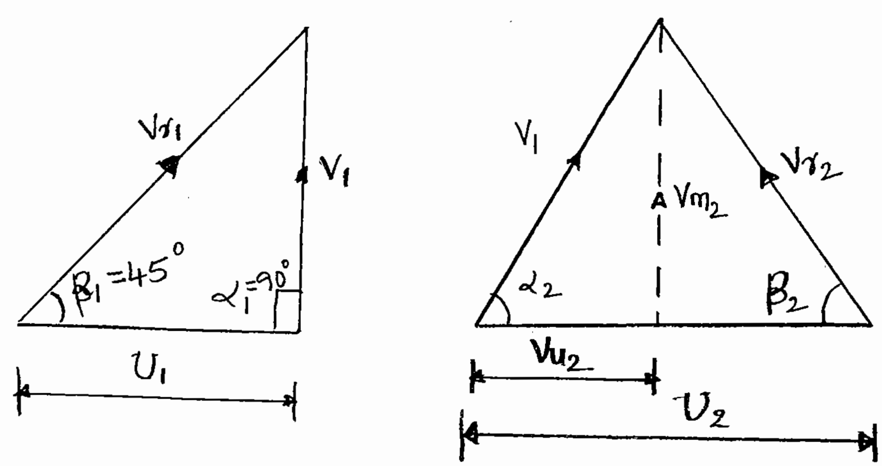
<u>Given</u>: Radial outward flow

$V_{u_1} = 0$, $\beta_1 = 45°$.

$U_2 = 2U_1$

$V_{m_1} = V_{m_2} = V_m$

V_{u_2} and U_2 are in same direction.



$V_1 = V_{m_1} = V_{m_2}$

$V_{u_2} \rightarrow$

$U_2 \rightarrow$

We have from Eulers energy equation

$$E = U_1 V_{u_1} - U_2 V_{u_2}.$$

$$E = -U_2 V_{u_2} \text{ --- } (1) \qquad \text{Since } V_{u_1} = 0$$

From outlet velocity Δ^{le}

$$\tan\beta_2 = \frac{V_m}{U_2 - V_{u_2}}$$

$$U_2 - V_{u_2} = V_m \cot\beta_2$$

$$V_{u_2} = U_2 - V_m \cot\beta_2 \text{ --- } (a)$$

From inlet velocity triangle.

$$\tan\beta_1 = \frac{V_m = V_1}{U_1}$$

$$\tan 45 = \frac{V_m}{U_1}$$

$$\boxed{V_m = U_1} \quad \text{also} \quad U_2 = 2U_1$$

$$(a) \Rightarrow \quad V_{u_2} = 2U_1 - U_1 \cot\beta_2 \text{ --- } (b)$$

Substitute (b) in (1)

$$(1) \Rightarrow \quad E = -U_2 [2U_1 - U_1 \cot\beta_2]$$

$$E = -2U_1 \cdot U_1 [2 - \cot\beta_2]$$

$$\boxed{E = -2U_1^2 (2 - \cot\beta_2)} \text{ --- } (2)$$

If β_2 increases, E decreases

We know

$$R = \frac{(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)} \quad \text{— (3)}$$

also $$E = \frac{1}{2}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]$$

but $$E = -2U_1^2(2 - \cot\beta_2) \qquad \text{from (2)}$$

$\Rightarrow$

$$(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2) = -4U_1^2(2 - \cot\beta_2) \quad \text{— (c)}$$

From inlet velocity triangle

$$V_{r_1}^2 = V_1^2 + U_1^2 \qquad \text{but } U_1 = V_1$$

$$V_{r_1}^2 = 2U_1^2 \quad \text{— (d)}$$

from outlet velocity triangle

$$V_{r_2}^2 = V_m^2 + (U_2 - V_{u_2})^2$$

$$= U_1^2 + (\cancel{2U_1^2}\ V_m\cot\beta_2)^2$$

$$= U_1^2 + U_1^2\cot^2\beta_2$$

$$V_{r_2}^2 = U_1^2(1 + \cot^2\beta_2) \quad \text{— (e)}$$

$$\Rightarrow \quad V_{r_2}^2 - V_{r_1}^2 = (U_1^2 + U_1^2\cot^2\beta_2) - (2U_1^2) \qquad \text{from (e) \& (d)}$$

$$V_{r_1}^2 - V_{r_1}^2 = -U_1^2 + U_1^2\cot^2\beta_2 \quad \text{— (f)}$$

Substitute (e) and (f) in (3)

(3) $\Rightarrow$

$$R = \frac{(u_1^2 - 2u_1^2) + (-u_1^2 + u_1^2\cot^2\beta_2)}{-4u_1^2(2-\cot\beta_2)}$$

$$= \frac{-4u_1^2 + u_1^2\cot^2\beta_2}{-4u_1^2(2-\cot\beta_2)} = \frac{-u_1^2(4-\cot^2\beta_2)}{-4u_1^2(2-\cot\beta_2)}$$

$$= \frac{(2+\cot\beta_2)(2-\cot\beta_2)}{4(2-\cot\beta_2)}$$

$$\boxed{R = \frac{2+\cot\beta_2}{4}} \quad \text{—(4)}$$

β_2 increase, R decreases

Effect of β_2 on E

It can be seen from (below) figure that for β_2 greater than about 30°, E is negative and continuously increases with β_2. But at $\beta_2 = 26.5°$, E becomes zero.

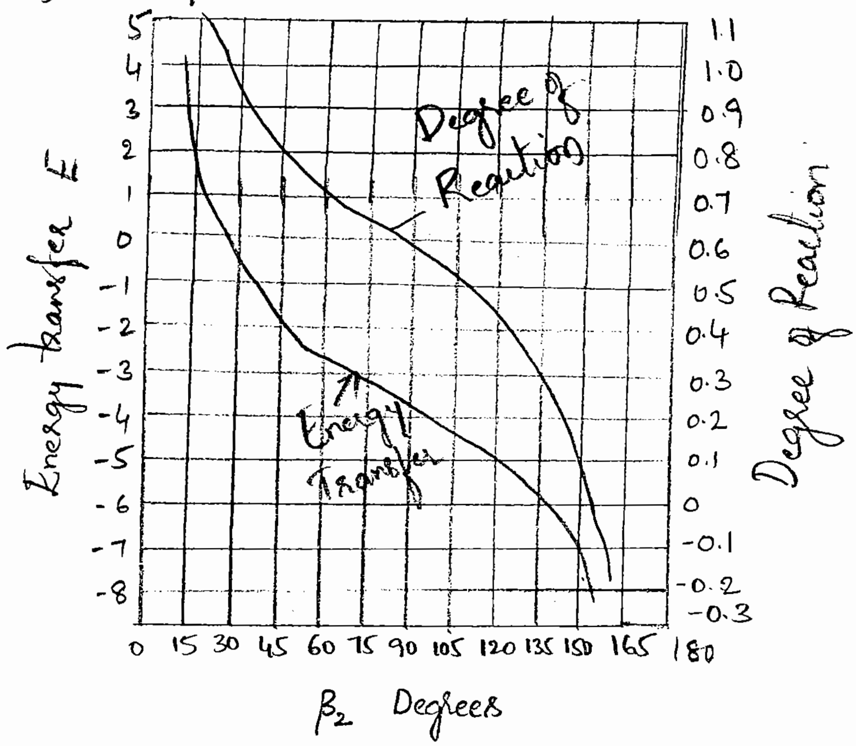
Effect of β_2 (Discharge angle) on 'R'

For β_2 in the range of 30–150°, the value of 'R' decreases linearly from near unity to very small positive value.

i) at $\beta_2 = 26.5°$, $R = 1$ and $E = 0$

ii) at $\beta_2 < 26.5°$ $R > 1$ and $E = +ve$ act as turbine

iii) at β_2 = 153.5   R = 0, act as impuls type



iv) at $\beta_2 > 26.5$ $R < 1$ and $E = -ve$ act as fan or pump, or compressor.

The above ~~at~~ analysis just shows that how a machine will act as a compressor or a turbine by varying one major variable β_2 only.

18. Show that E_{max} of an axial flow turbine with degree of reaction 0.25, the relationship of blades speed U to absolute velocity at rotor inlet V_1, should be $\frac{U}{V_1} = \frac{2}{3}\cos\alpha_1$, where α_1 is nozzle angle at inlet, assume flow velocity is constant from inlet to outlet.

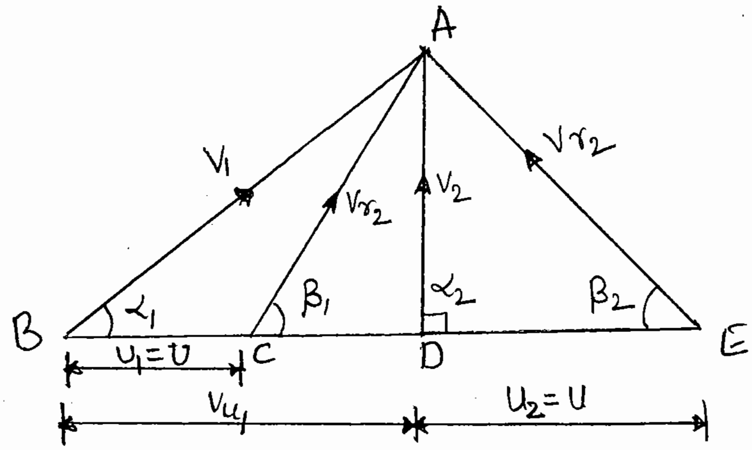
Given

Axial flow $U_1 = U_2 = U$

Degree of Reaction $R = 0.25 = \frac{1}{4}$

For maximum utilization factor. V_2 should be axial.

ie $V_{u_2} = 0$. , $V_{m_1} = V_{m_2} = V_m = V_2$



we have

$$R = \frac{(V_{r_2}^2 - V_{r_1}^2)}{(V_1^2 - V_2^2) + (V_{r_2}^2 - V_{r_1}^2)} = \frac{1}{4} \qquad \text{Since } U_1 = U_2$$

$$4(V_{r_2}^2 - V_{r_1}^2) = (V_1^2 - V_2^2) + (V_{r_2}^2 - V_{r_1}^2)$$

$$3(V_{r_2}^2 - V_{r_1}^2) = (V_1^2 - V_2^2) \quad \text{——} \quad (1)$$

From velocity triangle ABD.

$$\sin\alpha_1 = \frac{V_2}{V_1}$$

$$V_2 = V_1 \sin\alpha_1 \quad \text{——} \quad (a)$$

From Δ^{le} ADE

$$V_{r_2}^2 = V_2^2 + U^2$$

$$V_{r_2}^2 = V_1^2 \sin^2\alpha_1 + U^2 \quad \text{— (b)}$$

again from Δ^{le} ACD

$$V_{r_1}^2 = V_2^2 + CD^2$$

$$V_{r_1}^2 = V_2^2 + (V_{u_1} - U)^2$$

but $\cos\alpha_1 = \dfrac{V_{u_1}}{V_1}$, $V_{u_1} = V_1 \cos\alpha_1$

$$V_{r_1}^2 = V_1^2 \sin^2\alpha_1 + (V_1 \cos\alpha_1 - U)^2$$

$$V_{r_1}^2 = V_1^2 \sin^2\alpha_1 + V_1^2 \cos^2\alpha_1 + U^2 - 2UV_1\cos\alpha_1$$

$$V_{r_1}^2 = V_1^2 + U^2 - 2UV_1\cos\alpha_1 \quad \text{— (c)}$$

also $V_{r_2}^2 - V_{r_1}^2 = V_1^2 \sin^2\alpha_1 + U^2 - V_1^2 - U^2 + 2UV_1\cos\alpha_1$

$$= V_1^2(\sin^2\alpha_1 - 1) + 2UV_1\cos\alpha_1$$

$$V_{r_2}^2 - V_{r_1}^2 = 2UV_1\cos\alpha_1 - V_1^2\cos^2\alpha_1 \quad \text{— (2)}$$

also $V_1^2 - V_2^2 = V_1^2 - V_1^2\sin^2\alpha_1 = V_1^2\cos^2\alpha_1$ — (3)

put (2) and (3) in (1)

(1) $\Rightarrow$

$$3\left[2UV_1\cos\alpha_1 - V_1^2\cos^2\alpha_1\right] = V_1^2\cos^2\alpha_1$$

$$2U - V_1\cos\alpha_1 = \frac{V_1\cos\alpha_1}{3}$$

$$2U = \frac{4V_1\cos\alpha_1}{3}, \quad \Rightarrow \quad U = \frac{2V_1\cos\alpha_1}{3}$$

$$\boxed{\frac{U}{V_1} = \frac{2}{3}\cos\alpha_1}$$

STEAM TURBINE

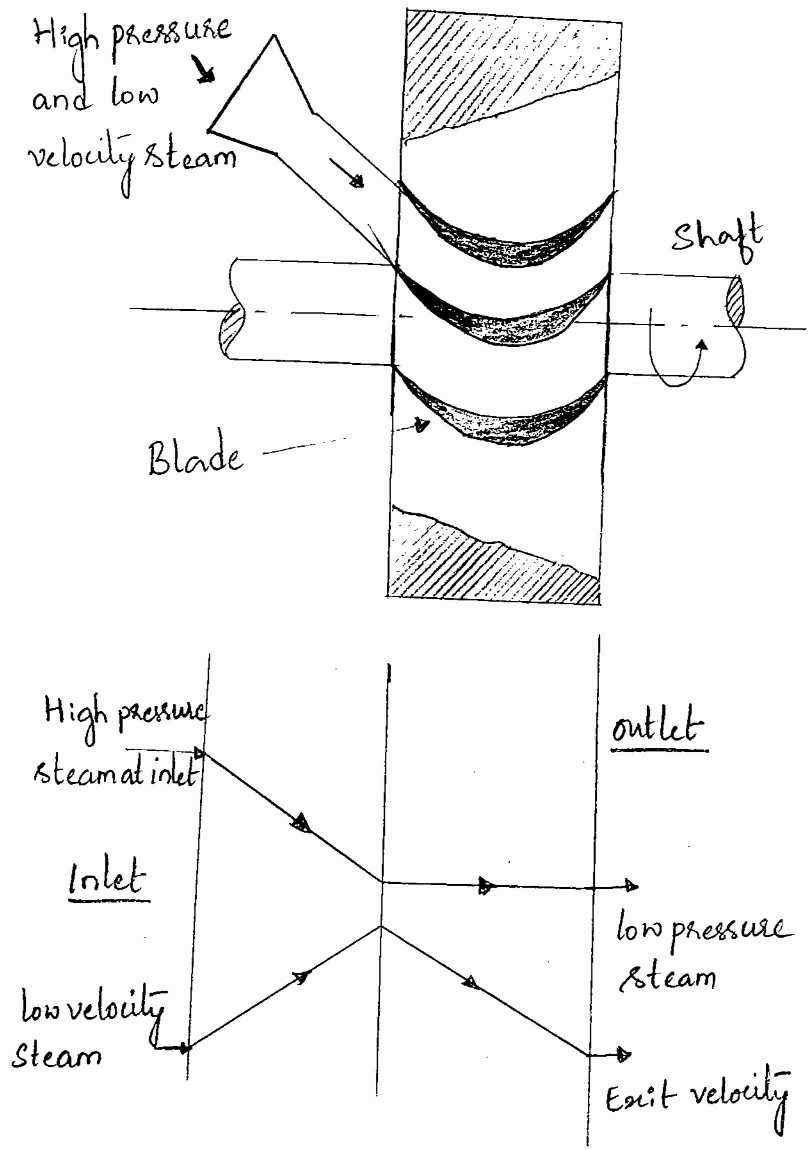
Steam turbines belongs to power generating turbo machines which uses the steam as a working fluid. High pressure steam from the boiler is expanded in nozzle, in which the enthalpy of steam is being converted into kinetic energy. Thus, the steam at high velocity at the exit of nozzle impinges over the moving blades which cause to change the flow direction of steam and thus cause a tangential force on the rotor blades.

Steam turbines may be of two kinds, namely

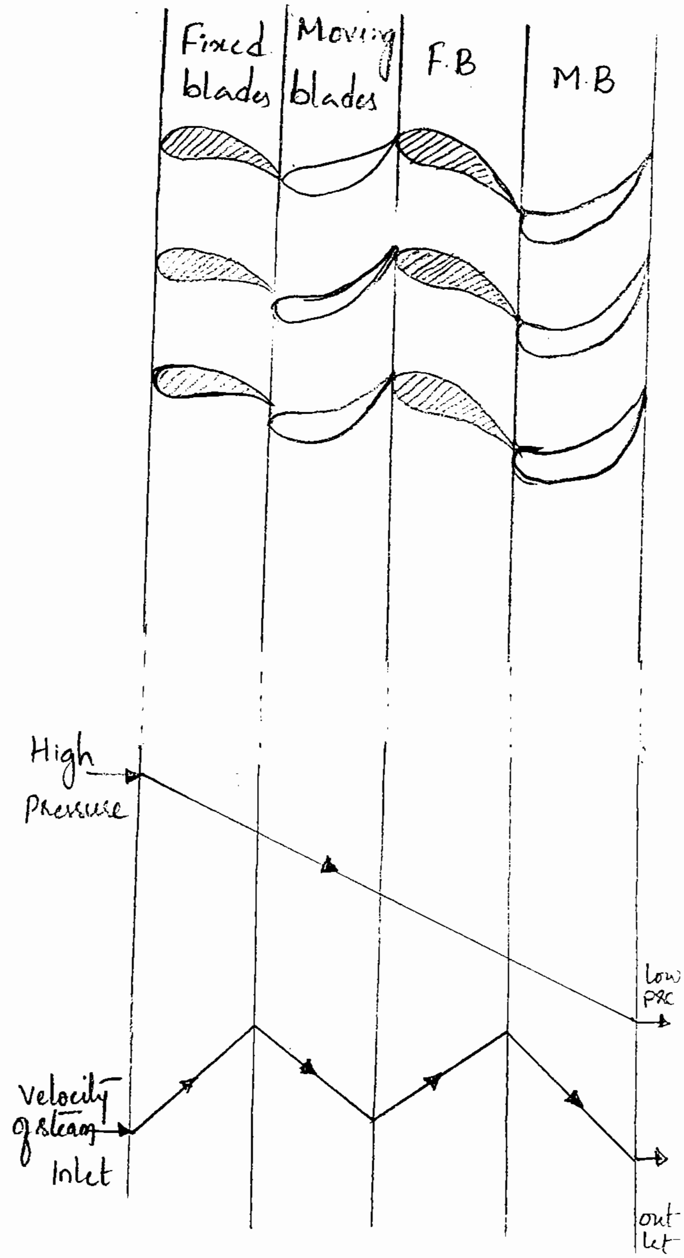
i) Impulse turbine.

ii) Reaction turbine.

In Impulse turbine, the enthalpy (pressure) drop completely occurs in the nozzle itself and when the fluid pass over the moving blades it will not suffer pressure drop again. Hence pressure remain constant when the fluid pass over the rotor blades. Figure 1 shows the schematic diagram of Impulse turbine. Ex: De laval, curtis, Reteau turbines, etc.



1. Impulse turbine



2. Reaction turbine.

In Reaction turbines, addition to the pressure drop occurs in the nozzle there will also be pressure drop occur when the fluid passes over the rotor blades. Fig 2 shows the diagram of reaction turbine

Example: parsons turbine, Ljungstrom turbine ....

Differences between Impulse and Reaction Steam Turbine

Sl No.	Particulars	Impulse turbine	Reaction turbine
01	Steam expansion	Expands fully in nozzle and pressure remain constant over the rotor blades.	Expands partially in nozzle and further expansion takes place when it passes over the rotor blades.
02	Relative velocity	Remains constant ie $V_{r_1} = V_{r_2}$	Go on increasing as the pressure drop occurs on the moving blades, ie $V_{r_2} > V_{r_1}$
03	Blade sections	Blades are symmetrical	Blades are of aero foil type.
04.	Steam velocity at the inlet of machine	very high	Moderate or low
05	Blade or stage efficiency	Comparatively low	High
06	Suitability	Suitable, where the efficiency is not a matter of fact	Suitable, where the efficiency is a matter of fact.

Compounding of Steam Turbine

Compounding can be defined as the method of obtaining reasonable tangential speed of rotor for given overall pressure drop by using more than one stages.

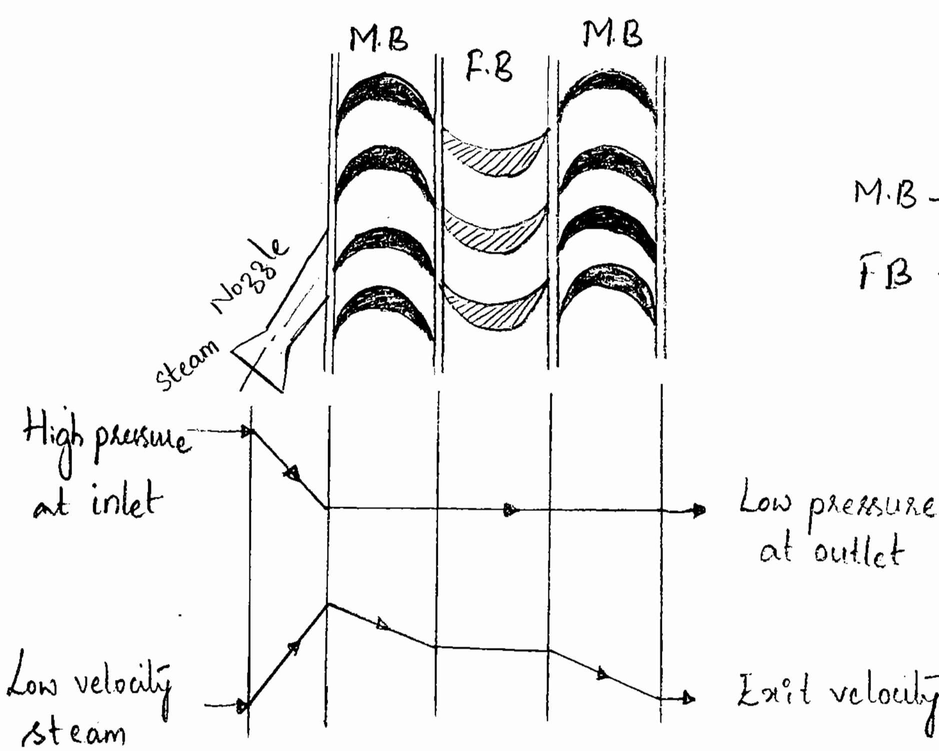
Compounding is necessary for steam turbines because if the tangential blade tip velocity greater than 400m/s, then the blade tips are subjected to centrifugal stress. Due this, utilization is low hence the efficiency of the stage is also low. compounding can be done by the following methods, namely

(i) Velocity compounding

(ii) Pressure compounding

(iii) Pressure-Velocity compounding

Velocity Compounding (Curtis stage) of Impulse turbine



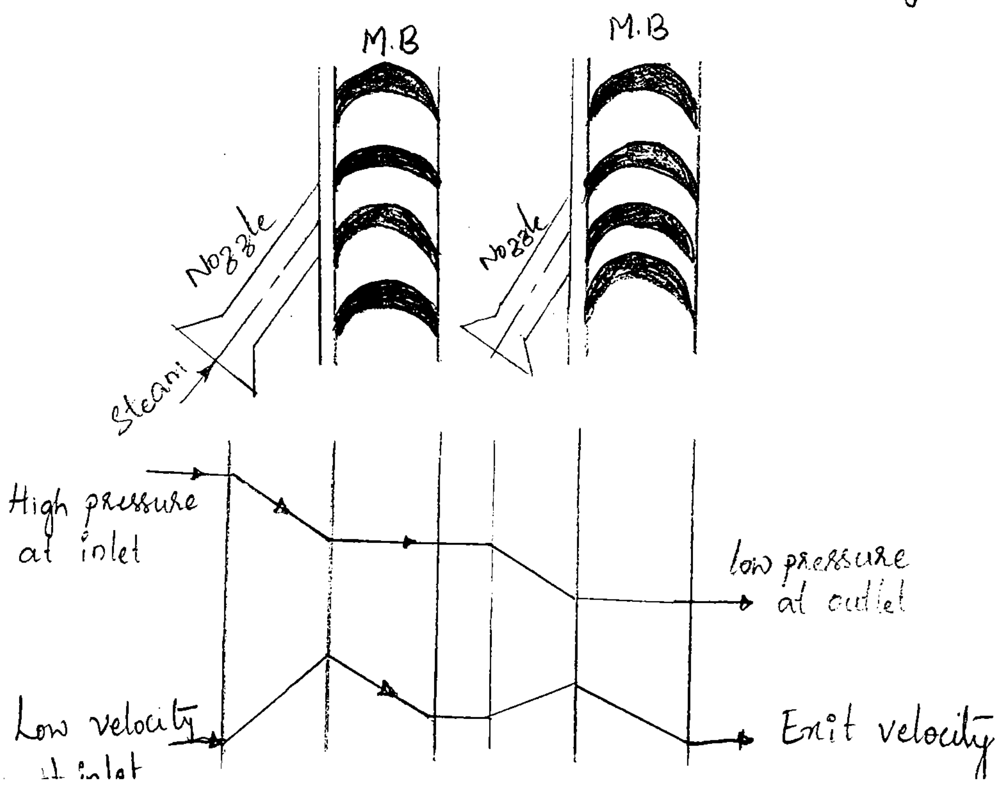
M.B – Moving blades

F.B - Fixed blades.

Velocity compounding consists of set of nozzles, rows of moving blades (rotor) and a rows of stationary blades (stator). Figure shows the corresponding velocity compounding impulse turbine. The function of stationary blades is to direct the steam coming from the first moving row to the next moving row without appreciable change in velocity. All the kinetic energy available at the nozzle exit is successively absorbed by all the moving rows and the steam is sent from the last moving row with low velocity to achieve high utilization. The turbine works under this type of compounding stage is called velocity compounded turbine.

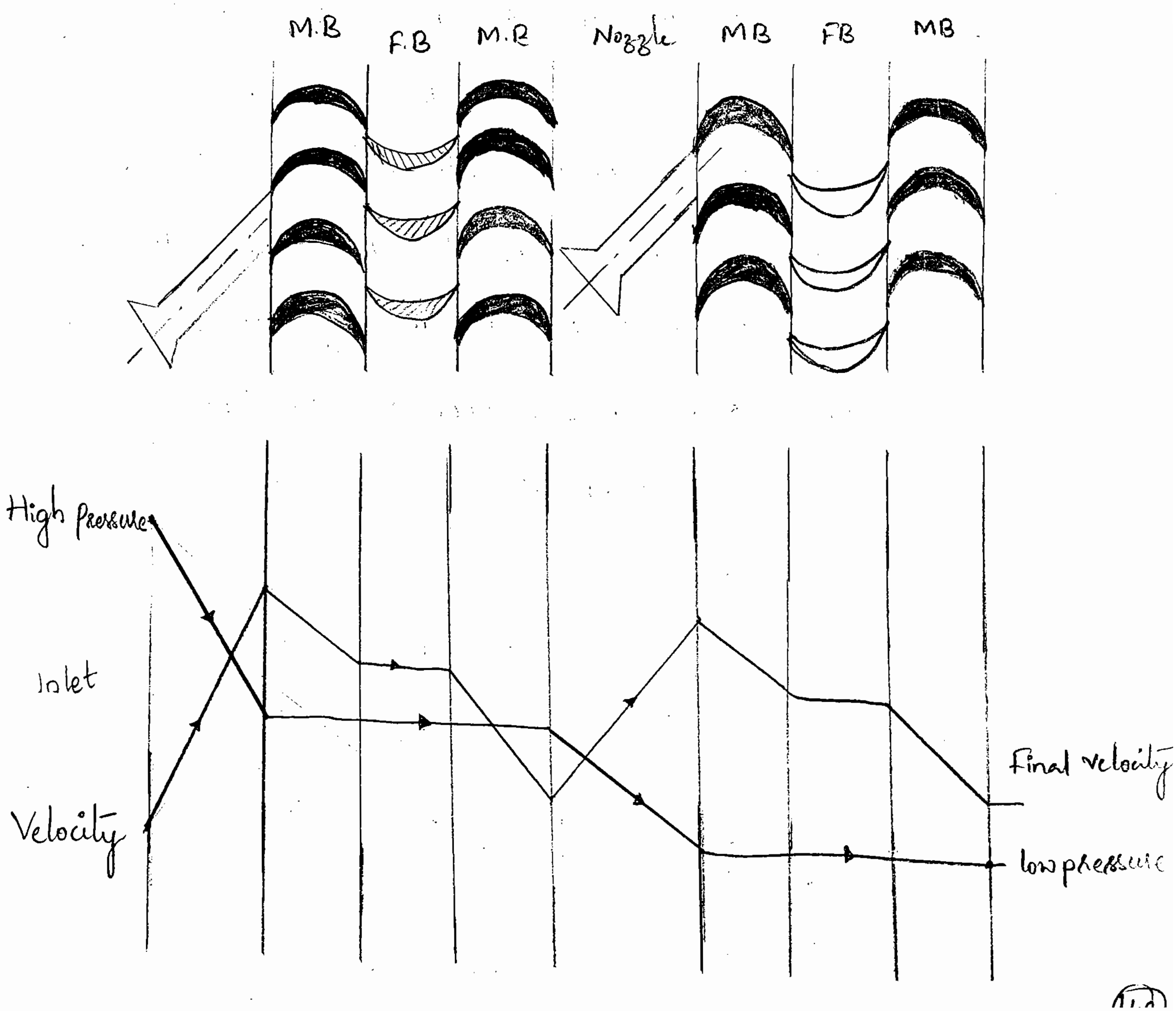
example: Curtis stage steam turbine.

Pressure compounded (Rateau Stage) Impulse turbine



A number of simple impulse stages arranged in series is called as pressure compounding. In this case, the turbine is provided with rows of fixed blades which acts as a nozzles at the entry of each rows of moving blades. The total pressure drop of steam does not take place in a single nozzle but divided among all the rows of ~~ft~~ fixed blades which act as nozzle for the next moving rows.

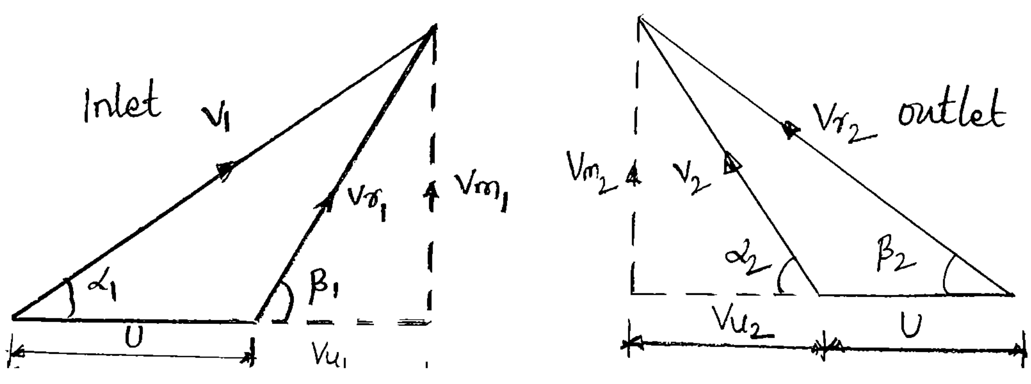
Pressure-Velocity compounding



In this method high rotor speeds are reduced without sacrificing the efficiency or the output. pressure drop from the chest pressure to the condenser pressure occurs at two stages. This type of arrangement is very popular due to simple construction as compared to pressure compounding steam turbine.

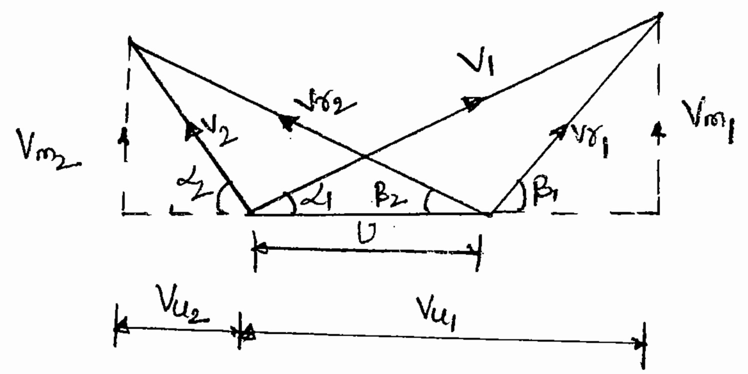
It consists of a set of nozzles and rows of moving blades fixed to the shaft and rows of fixed blades to casing at each stage. The entire expansion takes place in the nozzles. The high velocity steam parts with only portion of the kinetic energy in the first set of moving blades and then passed on to fixed blades where only change in direction of jets takes place without appreciable loss in velocity. This jet then passes on to another set of moving vanes where further drop in kinetic energy occurs. This type of turbine is also called curtis turbine.

Analysis of Single stage Impulse Turbine



Since 'U' is constant, we can combined inlet and outlet velocity triangles

ie



(1) Tangential force

$$F_T = \frac{\dot{m}}{g_c}\left[V_{u_1} \pm V_{u_2}\right] = \text{-----}\ N$$

(2) Axial thrust

$$F_a = \frac{\dot{m}}{g_c}\left(V_{m_1} - V_{m_2}\right] = \text{-----}\ N$$

(3) Blade efficiency or Diagram efficiency

It is defined as the ratio of work done per kg of steam by the rotor to the energy available at the inlet per kg of steam ie

$$\eta_b = \frac{\text{work done/kg of steam by the rotor}}{\text{Energy available/kg of steam at inlet.}}$$

$$\eta_b = \frac{E}{(V_1^2/2)} = \frac{U[V_{u1} \pm V_{u2}]}{(V_1^2/2)}$$

4) <u>Stage efficiency</u> (η_s)

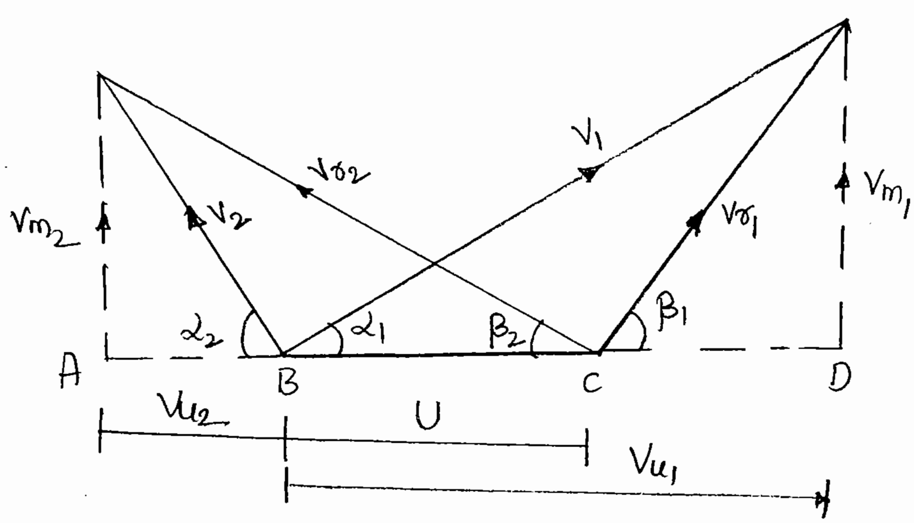
It is defined as "the ratio of work done per kg of steam by the rotor to the isentropic enthalpy change per kg of steam in the nozzle".

$$\therefore \quad \eta_s = \frac{E}{\Delta h'} = \frac{E}{(V_1^2/2)} \cdot \frac{(V_1^2/2)}{\Delta h'}$$

$$\eta_s = \eta_b \cdot \eta_n \qquad \text{where } \eta_n = \frac{V_1^2/2}{\Delta h'}, \text{ nozzle efficiency.}$$

Condition for Maximum utilization factor or Blade efficiency with equiangular Blades for impulse turbine:

$\boxed{\text{OR}}$ To prove $\eta_{b\,max} = \cos^2\alpha_1$



Due to the effect of blade friction loss, the relative velocity at outlet is reduced than the relative velocity at inlet

ie $\boxed{C_b = V_{r_2}/V_{r_1}}$

we know, Energy transfer $E = \dfrac{U[V_{u_1} + V_{u_2}]}{g_c}$ — (1)

From velocity triangle, $V_{u_1} + V_{u_2} = AB + BD$

also $V_{u_1} + V_{u_2} = AC + CD$

$$= V_{r_2}\cos\beta_2 + V_{r_1}\cos\beta_1$$

$$V_{u_1} + V_{u_2} = V_{r_1}\cos\beta_1\left(1 + \frac{V_{r_2}}{V_{r_1}}\frac{\cos\beta_2}{\cos\beta_1}\right) \quad \text{— (2)}$$

also $V_{r_1}\cos\beta_1 = V_1\cos\alpha_1 - U$

hence (2) $\Rightarrow$

$$V_{u_1} + V_{u_2} = (V_1\cos\alpha_1 - U)\left(1 + \frac{V_{r_2}}{V_{r_1}}\frac{\cos\beta_2}{\cos\beta_1}\right)$$

but $\dfrac{V_{r_2}}{V_{r_1}} = C_b$ and let $k = \cos\beta_2/\cos\beta_1$

$$V_{u_1} + V_{u_2} = (V_1\cos\alpha_1 - U)(1 + C_b k)$$

but energy $E = U[V_{u_1} + V_{u_2}] = U(V_1\cos\alpha_1 - U)(1 + C_b k)$

$$E = \frac{UV_1^2}{V_1}\left(\cos\alpha_1 - \frac{U}{V_1}\right)(1 + C_b k) \quad \text{— (3)}$$

but we know $\frac{U}{V_1} = \varphi$

(3) $\Rightarrow$ $$E = \frac{UV_1}{V_1} . V_1 \left(\cos\alpha_1 - \frac{U}{V_1}\right) \left(1 + C_b K\right)$$

$$E = V_1^2 \phi \left(\cos\alpha_1 - \varphi\right) \left(1 + C_b K\right) \quad \text{--- (4)}$$

also available energy at inlet is $(V_1^2/2)$

we know,

Blade efficiency $\eta_b = \frac{E}{(V_1^2/2)}$

$$= \frac{V_1^2 \phi (\cos\alpha_1 - \phi)(1 + C_b K)}{(V_1^2/2)}$$

$$\eta_b = 2[\phi \cos\alpha_1 - \varphi^2][1 + C_b K] \quad \text{--- (5)}$$

For maximum blade efficiency $\frac{d\eta_b}{d\phi} = 0$

ie $\frac{d}{d\phi} 2(\phi\cos\alpha_1 - \phi^2)(1 + C_b K) = 0$

$$\cos\alpha_1 - 2\phi = 0$$

$$\phi_{opt} = \frac{\cos\alpha_1}{2} = \text{speed ratio}$$

(5) $\Rightarrow$

$$\eta_{b_{max}} = 2\left[\frac{\cos\alpha_1}{2} . \cos\alpha_1 - \frac{\cos^2\alpha_1}{4}\right] (1 + C_b K)$$

$$\eta_b = \frac{\cos^2\alpha_1}{2}(1 + C_b k)$$

if rotor angles are equiangulare ie $\beta_1 = \beta_2$ and $V_{r_1} = V_{r_2}$

$$C_b = \frac{V_{r_2}}{V_{r_1}} = 1, \quad k = \frac{\cos\beta_2}{\cos\beta_1} = 1$$

$$\eta_{b_{max}} = \frac{\cos^2\alpha_1}{2}(1 + 1.1)$$

$$\boxed{\eta_{b_{max}} = \cos^2\alpha_1}$$

Problems on single stage

1) In a Delaval turbine, steam flows from a nozzle with a velocity of 1200m/s. The nozzle angle is 20°. The mean blade speed is 400m/s and inlet and outlet angles of blades are equal. The mass of steam flowing through the turbine is 1000kg/hr. Calculate

a) Blade angle at inlet and outlet.

b) Relative velocity of steam entering the blades.

c) Tangential force on the blades.

d) power developed and Blade efficiency

e) Axial thrust, Take blade velocity coefficient as 0.8

Given:

Delaval turbine.

$V_1 = 1200 m/s$, $\alpha_1 = 20°$

$u = 400 m/s$, $\beta_1 = \beta_2$

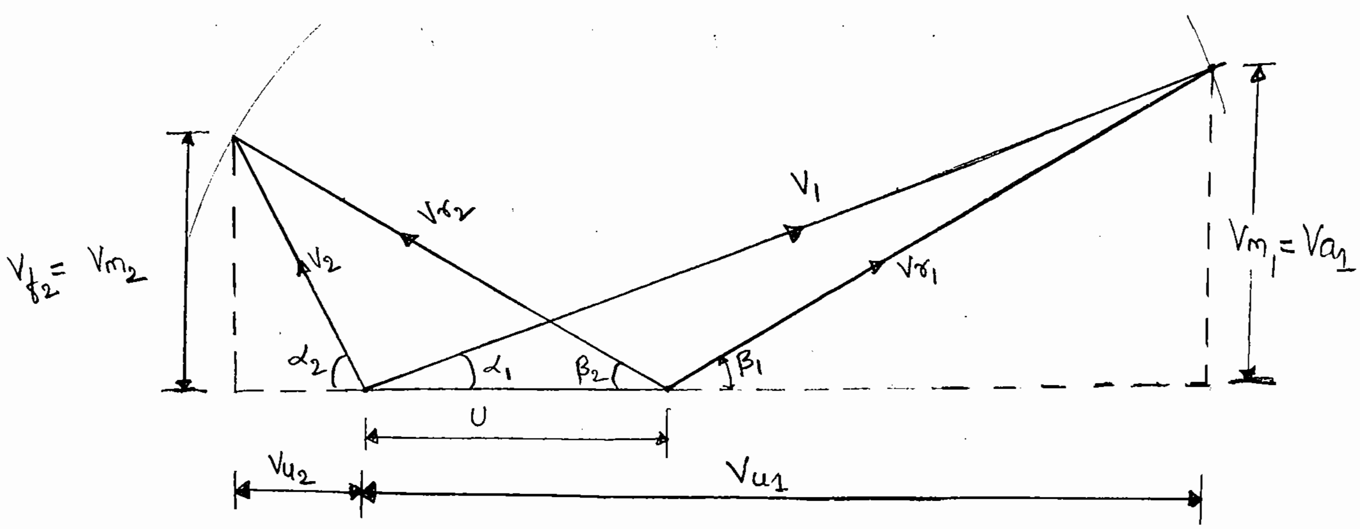
$\dot{m} = 1000 kg/hr$

$C_b = \frac{V_{r2}}{V_{r1}} = 0.8$

To find: $\beta_1, \beta_2, V_{r1}, F_T, P, F_a, \eta_b$

Scale $1 cm = \cancel{16 m/s}\ 100 m/s$

$\Rightarrow$ $V_1 = 7.5 cm$, $U = 2.5 cm$



From graph

$V_{r_1} = 8.4\,cm = 840\,m/s$

Relative velocity at inlet $\boxed{V_{r_1} = 840\,m/s}$

β_1 and β_2 are equal to 30°

ie $\boxed{\beta_1 = \beta_2 = 30^\circ}$

also

$V_{u_1} = 11.2\,cm = 1120\,m/s$

$V_{u_2} = 1.7\,cm = 170\,m/s$

$V_{a_1} = V_{m_1} = V_{f1} = 4.2\,cm = 420\,m/s$

$V_{a_2} = V_{m_2} = V_{f2} = 3.4\,cm = 340\,m/s$

we know,

Tangential force $F_T = \dot{m}\left[V_{u_1} + V_{u_2}\right]$, Since $V_{u_1} \longrightarrow$ $V_{u_2} \longleftarrow$

$$F_T = \frac{1000}{3600}\left[1120 + 170\right]$$

$$\boxed{F_T = 358.33\,N}$$

power $P = \dot{m}\,U\left[V_{u_1} + V_{u_2}\right]$

$$= \frac{1000}{3600} \times 400\left[1120 + 170\right] = 143333.33$$

$$\boxed{P = 143.3\,KW}$$

Axial thrust $F_a = \dot{m}\left[V_{m_1} - V_{m_2}\right]$

$$= \frac{1000}{3600}\left[420 - 340\right]$$

$$\boxed{F_a = 22.22\,N}$$

Diagram or Blade efficiency $\eta_b = \frac{E}{(V_1^2/2)}$

$$= \frac{U\left[V_{u_1} + V_{u_2}\right]}{(V_1^2/2)} = \frac{400(1120+170)}{(1200^2/2)}$$

$$= 0.71666.$$

$$\boxed{\eta_b = 71.667\,\%.}$$

2. A Single stage impulse turbine rotor has a diameter of 1.2m running at 3000rpm. The nozzle angle is 18°. Blade speed ratio is 0.42. The ratio of relative velocity at outlet to that at inlet is 0.9. The outlet angle of the blade is 3° less than the inlet angle. Steam flow rate is 5kg/s. Draw the velocity triangles and find a) Velocity of whirl

b) Axial thrust on Bearing

c) Blade angles.

d) Power developed.

Given:

Diameter $D = 1.2m$

Speed $N = 3000 rpm$

$\beta_2 = \beta_1 - 3^\circ$, $\alpha_1 = 18^\circ$

ϕ = Speed ratio $= \frac{U}{V_1} = 0.42$

$C_b = \frac{V_{r_2}}{V_{r_1}} = 0.9$, $\dot{m} = 5 kg/s$

To find: $V_{u_1}, V_{u_2}, F_a, \beta_1, \beta_2, P$

we know $U = \frac{\pi DN}{60} = \frac{\pi \times 1.2 \times 3000}{60}$

$U = 188.495 \simeq 188.5 m/s$

$\boxed{U = 188.5 m/s}$

also

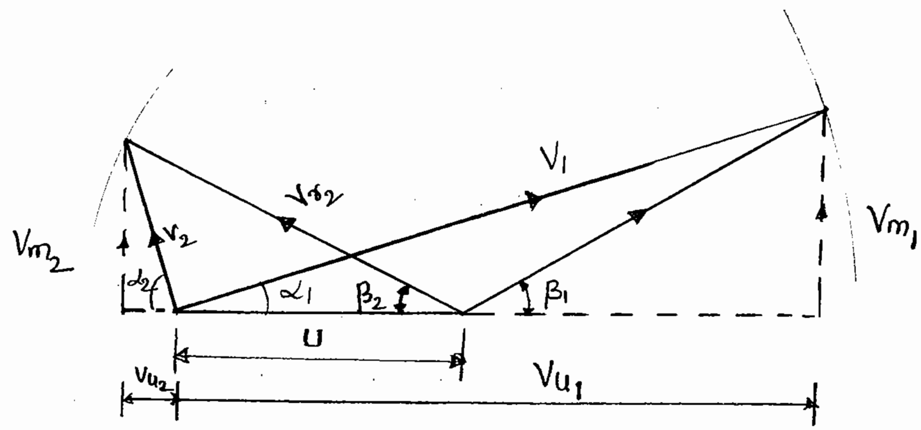
$\phi = \frac{U}{V_1} = 0.42 \Rightarrow V_1 = \frac{188.5}{0.42}$

$\boxed{V_1 = 448.8 m/s}$

Scale 1 cm = 50 m/s.

$\Rightarrow$ $U = 3.77\text{cm} \simeq 3.8\text{cm}$

$V_1 = 8.976\text{cm} \simeq 8.9\text{cm}$



From graph

$V_{r_1} = 5.5$ cm,

$$C_b = \frac{V_{r_2}}{V_{r_1}} = 0.9$$

$V_{r_2} = 0.9 \times 5.5 = 4.95 \simeq 5\text{cm}$.

$\boxed{\beta_1 = 30^\circ}$ and $\boxed{\beta_2 = 27^\circ}$

$V_{u_1} = 8.5\text{cm} = 8.5 \times 50 = \underline{\underline{425}}$ m/s

$V_{u_2} = 0.7\text{cm} = 0.7 \times 50 = \underline{\underline{35}}$ m/s

$V_{m_1} = 2.7\text{cm} = 2.7 \times 50 = \underline{\underline{135}}$ m/s

$V_{m_2} = 2.2\text{ cm} = 2.2 \times 50 = 110$ m/s

We know,

Axial thrust $F_a = \dot{m}(V_{m_1} - V_{m_2})$

$$= 5[135 - 110]$$

$$\boxed{F_a = 125\,N}$$

Power, $P = \dot{m}E = \dot{m}U[V_{u_1} + V_{u_2}]$

$$= 5 \times 188.5[425 + 35]$$

$$\boxed{P = 433.55\,kW}$$

3. One stage of an impulse turbine consists of a nozzle and one ring of moving blades. The nozzle is inclined at 22° to the tangential speed of blades and the blade tip angles are equiangular and equal to 35°.

a) Find the blade speed, diagram efficiency by neglecting losses, if the velocity of steam at the exit of the nozzle is 660 m/s.

b) If the relative velocity of steam is reduced by 15% in passing through the blade ring. Find the diagram efficiency and end thrust on the shaft when the blade ring develops 1745 kW.

Given:

One stage impulse steam turbine.

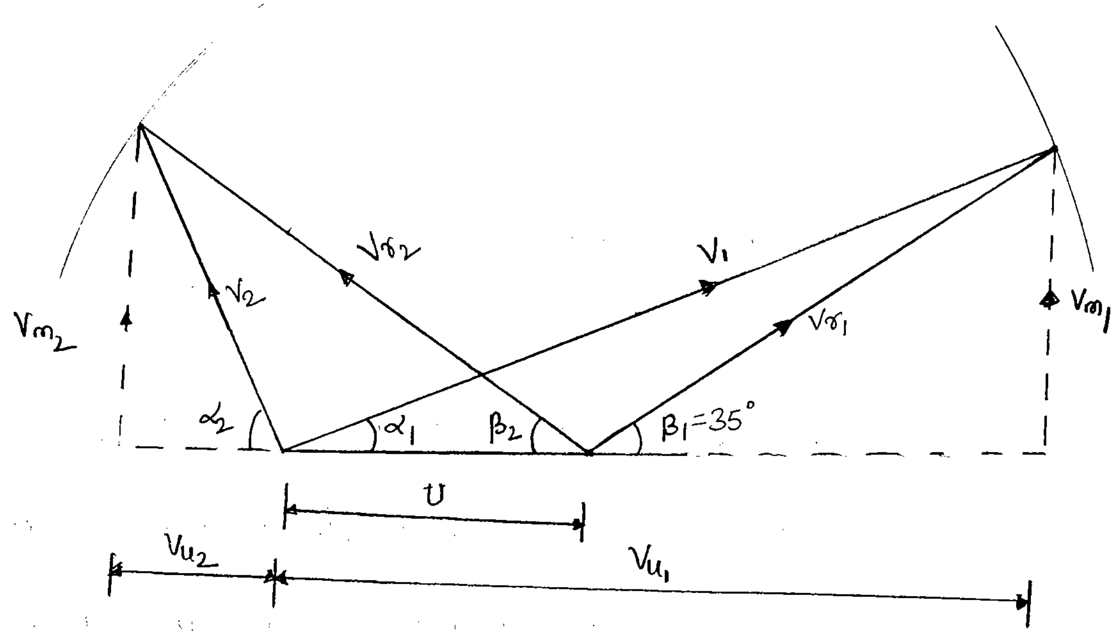
$\alpha_1 = 22^\circ$, $\beta_1 = \beta_2 = 35^\circ$

$V_1 = 660\,m/s$

case b) $C_b = 0.85$, $P = 1745\,kW$.

To find: U, η_b, η_b, F_a

<u>case a)</u> when there is no loss ie $V_{r_1} = V_{r_2}$.



From graph.

$$U = 4.2\,cm = 4.2 \times 60 = 252\,m/s$$

Tangential speed of rotor $\boxed{U = 246\,m/s}$

$$V_{r_1} = 7.2\,cm = 432\,m/s = V_{r_2}.$$

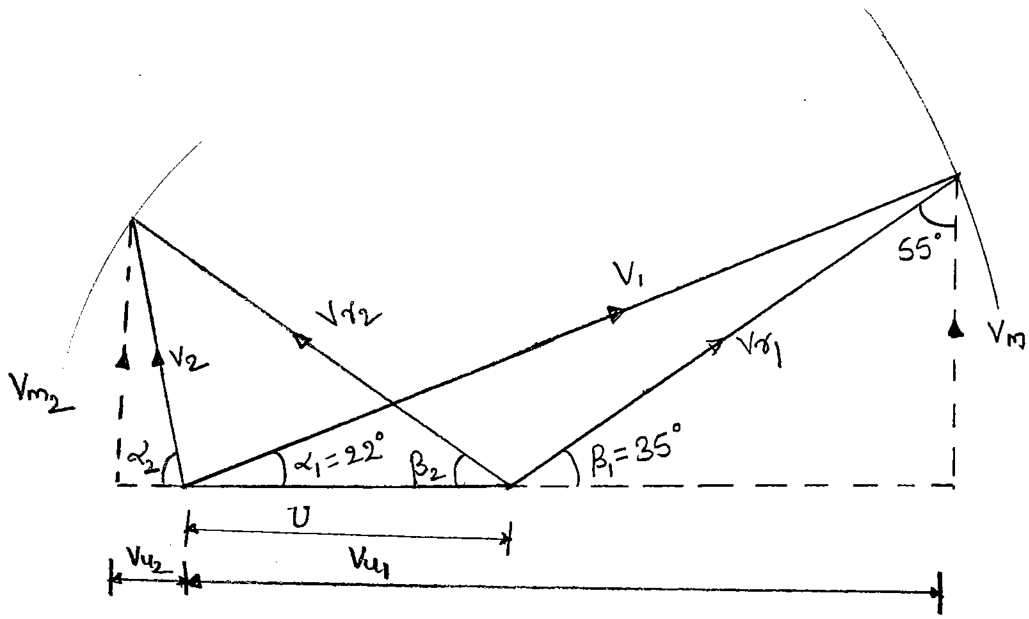
$$V_{u_1} = 10.1\,cm = 606\,m/s$$

$$V_{u_2} = 2.2\,cm = 132\,m/s$$

Blade efficiency $\eta_b = \dfrac{U[V_{u_1} + V_{u_2}]}{(V_1^2/2)} = \dfrac{252[606+132]}{(660^2/2)}$

$$\eta_b = \underline{83.36\,\%} \quad (\underline{85.4\,\%})$$

Case b) $V_{r_2} = 0.85 V_{r_1}$ ie $C_b = 0.85$



From graph

$V_{r_1} = 7.2\,cm = 7.2 \times 60 = 432\,m/s$

$C_b = \dfrac{V_{r_2}}{V_{r_1}} = 0.85 \Rightarrow V_{r_2} = 367.2\,m/s$

or $V_{r_2} = 6.1\,cm$

$V_{u_1} = 10.1\,cm = 606\,m/s$

$V_{u_2} = 0.9\,cm = 54\,m/s$

$V_{m_1} = 4.1\,cm = 246\,m/s$

$V_{m_2} = 3.5\,cm = 210\,m/s$

we know.

Blade efficiency $\eta_b = \dfrac{U[V_{u_1} + V_{u_2}]}{(V_1^2/2)}$

$U = 4.3\text{cm} = 258 \text{ m/s}$.

$$\eta_b = \frac{258[606+54]}{(660^2/2)}$$

$$\boxed{\eta_b = 78.18\%}$$

also power $= \dot{m}\, u[V_{u_1} + V_{u_2}]$

$$1745 = \dot{m}\; 258[606+54]$$

$$\underline{\dot{m} = 10.248 \text{ kg/s}}$$

Axial thrust $F_a = \dot{m}[V_{m_1} - V_{m_2}]$

$$= 10.248[246 - 210]$$

$$\boxed{F_a = 368.93 \text{ N}}$$

4. Dry saturated steam at 10 atmospheric pressure is supplied to single rotor impulse wheel, the condenser pressure being 0.5 atmosphere with the nozzle efficiency of 0.94 and the nozzle angle at the rotor inlet is 18° to the wheel plane. The rotor blades which moves with the speed of 450 m/s are equiangular. If the co-efficient of velocity for the rotor blades is 0.92, find

i) The specific power output

ii) The rotor efficiency, stage efficiency

iii) Axial thrust

iv) The direction of exit steam.

Given:

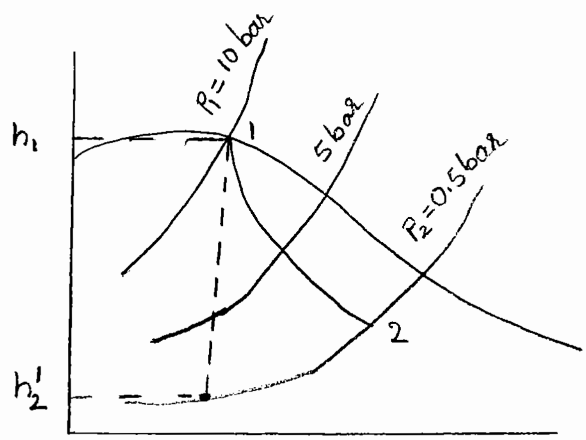
$P_1 = 10$ atm , $P_2 = 0.5$ atm

$\eta_n = 0.94$, $\alpha_1 = 18°$

$U = 450$ m/s , $\beta_1 = \beta_2$

$C_b = \dfrac{V_{r_2}}{V_{r_1}} = 0.92$

To find P, η_b, η_s, F_a, α_2



From Mollier chart

at $P_1 = 10$ bar, $h_1 = 2780$ kJ/kg

at $P_2 = 0.5$ bar, $h_2' = 2290$ kJ/kg

also we know nozzle efficiency $\eta_n = \dfrac{\text{Actual change in enthalpy}}{\text{Isentropic change in enthalpy}}$

ie $\eta_n = \dfrac{(V_1^2/2)}{h_1 - h_2'} = \dfrac{h_1 - h_2}{h_1 - h_2'}$

$\Rightarrow$

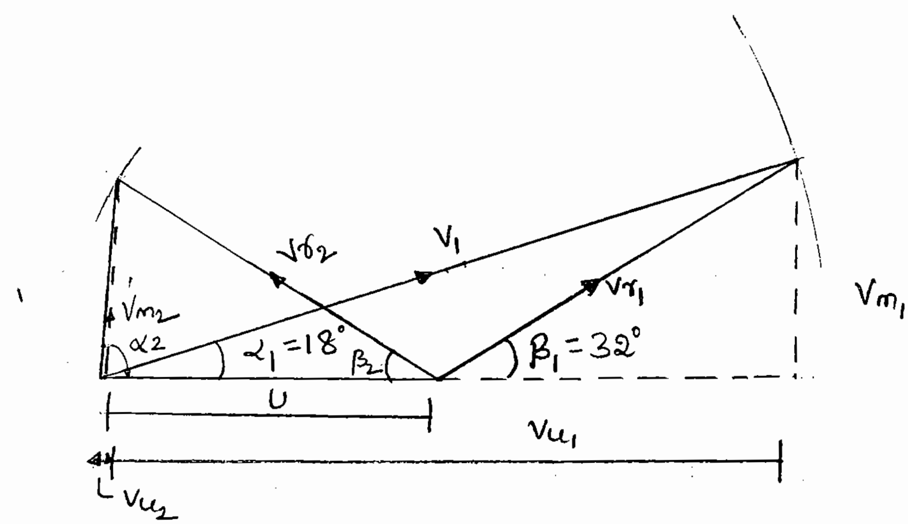
$$0.94 = \frac{(V_1^2/2)}{(2780-2290)\times 10^3}$$

$$10^3 \times (460.6) = \frac{V_1^2}{2}$$

$$V_1^2 = 2 \times 10^3 \times 460.6$$

$$\boxed{V_1 = 959.79\ m/s}$$

Scale 1cm = 100m/s



From graph:

$\boxed{\alpha_2 = 89°}$ direction of exit steam.

$V_{u_1} = 9.2cm = 920\ m/s$

$V_{u_2} = 0.05cm = 5\ m/s$

$V_{m_1} = 2.8 cm = 280 m/s$

$V_{m_2} = 26 cm = 260 m/s$

i) power $P = \dot{m}E = \dot{m}U[V_{u_1} \pm V_{u_2}]$

$$\frac{P}{\dot{m}} = 450[920 - 5]$$

$$\frac{P}{\dot{m}} = \underline{\underline{411.75}}\ kw/(kg/s)$$

ii) Rotor efficiency $\eta_r = \frac{E}{(V_1^2/2)} = \frac{U[V_{u_1} - V_{u_2}]}{(V_1^2/2)}$

$$= \frac{450(920-5)}{(959.79^2/2)}$$

$\eta_r = 89.39\%$

iii) Stage efficiency $\eta_s = \eta_n \times \eta_r = 0.94 \times 0.8939$

$\eta_s = 0.84027$

$\boxed{\eta_s = 84.03\%}$

iv) Axial thrust $F_a = \dot{m}[V_{m_1} - V_{m_2}]$

$$\frac{F_a}{\dot{m}} = [280 - 260]$$

$$\frac{F_a}{\dot{m}} = \underline{\underline{20}}\ N/(kg/s)$$

5. A Single stage impulse wheel is supplied with super heated steam at 1.5 MPa and 200°c, expands to 0.05 MPa condensor pressure. The rotors are fitted with equiangular blades moving at 450 m/s. of the nozzle angle at the rotor inlet is 16° to the wheel plane. Find the specific power output, blade efficiency, gross-stage efficiency, axial thrust and direction of exit steam velocity. Assume nozzle efficiency as 94% and relative velocities are equal.

Given

$P_1 = 1.5\,MPa = 15\,bar \quad T_1 = 200°c$

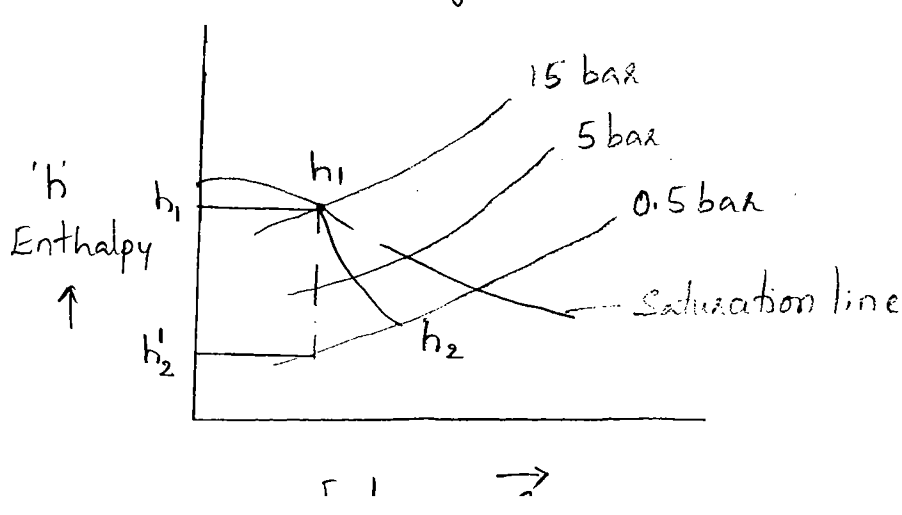
$P_2 = 0.05\,MPa = 0.5\,bar$

$\beta_1 = \beta_2$, Blades are equiangular

$U = 450\,m/s, \; \alpha_1 = 16°, \; \eta_n = 0.94$

To find: $\frac{P}{\dot{m}}, \; \eta_b, \; \eta_s, \; F_a, \; \alpha_2$ [ie direction of V_2]

From Mollier diagram



$h_1 = 2790 \ kJ/kg$

$h_2' = 2240 \ kJ/kg$

we know $\quad \eta_n = \dfrac{\text{Actual change in enthalpy in nozzle}}{\text{Isentropic change in enthalpy}}$

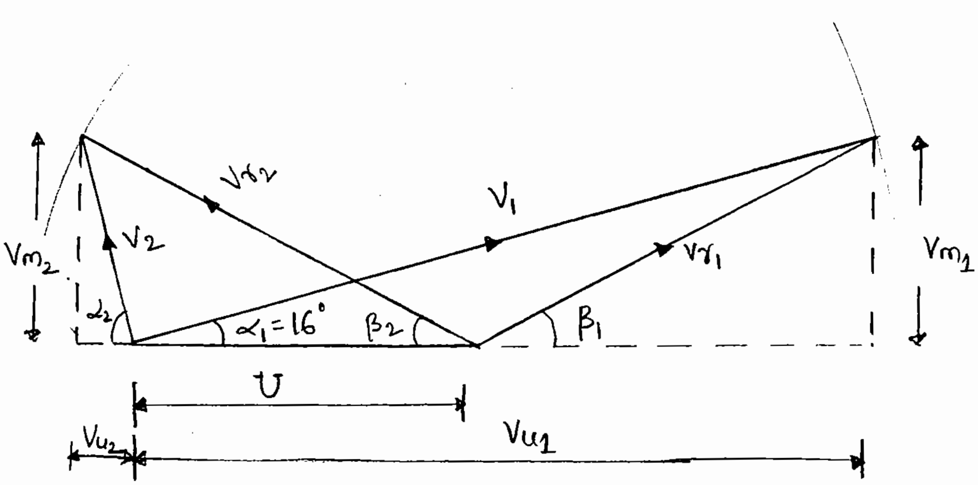
$$\eta_n = \frac{h_1 - h_2}{h_1 - h_2'} = \frac{(V_1^2/2)}{h_1 - h_2'}$$

$$0.94 = \frac{(V_1^2/2)}{(2790 - 2240) \times 10^3}$$

$$\boxed{V_1 = 1016.86 \ m/s} \simeq 1020 \ m/s$$

Scale $\quad 1cm = 100 m/s$

$\Rightarrow \quad U = 4.5cm, \ V_1 = 10.168 \simeq 10.2cm \ (1020 m/s)$



from graph.

$$V_{u_1}+V_{u_2} = 10.5\text{cm} = 10.5\times 100 = 1050\ \text{m/s}$$

Since $V_{r_1}=V_{r_2}$ and $\beta_1=\beta_2 \Rightarrow V_{m_1}=V_{m_2}=2.8\text{cm} = 280\ \text{m/s}$

we know,

$$\eta_b = \frac{E}{V_1^2/2} = \frac{U[V_{u_1}+V_{u_2}]}{(V_1^2/2)}\times 100$$

$$= \frac{450\times 1050}{(1020^2/2)}$$

$$\boxed{\eta_b = 90.83\%}$$

Gross efficiency $\eta_s = \eta_n \cdot \eta_b$

$$= 0.94\times 0.9083$$

$$\eta_s = 0.8538$$

$$\boxed{\eta_s = 85.38\%}$$

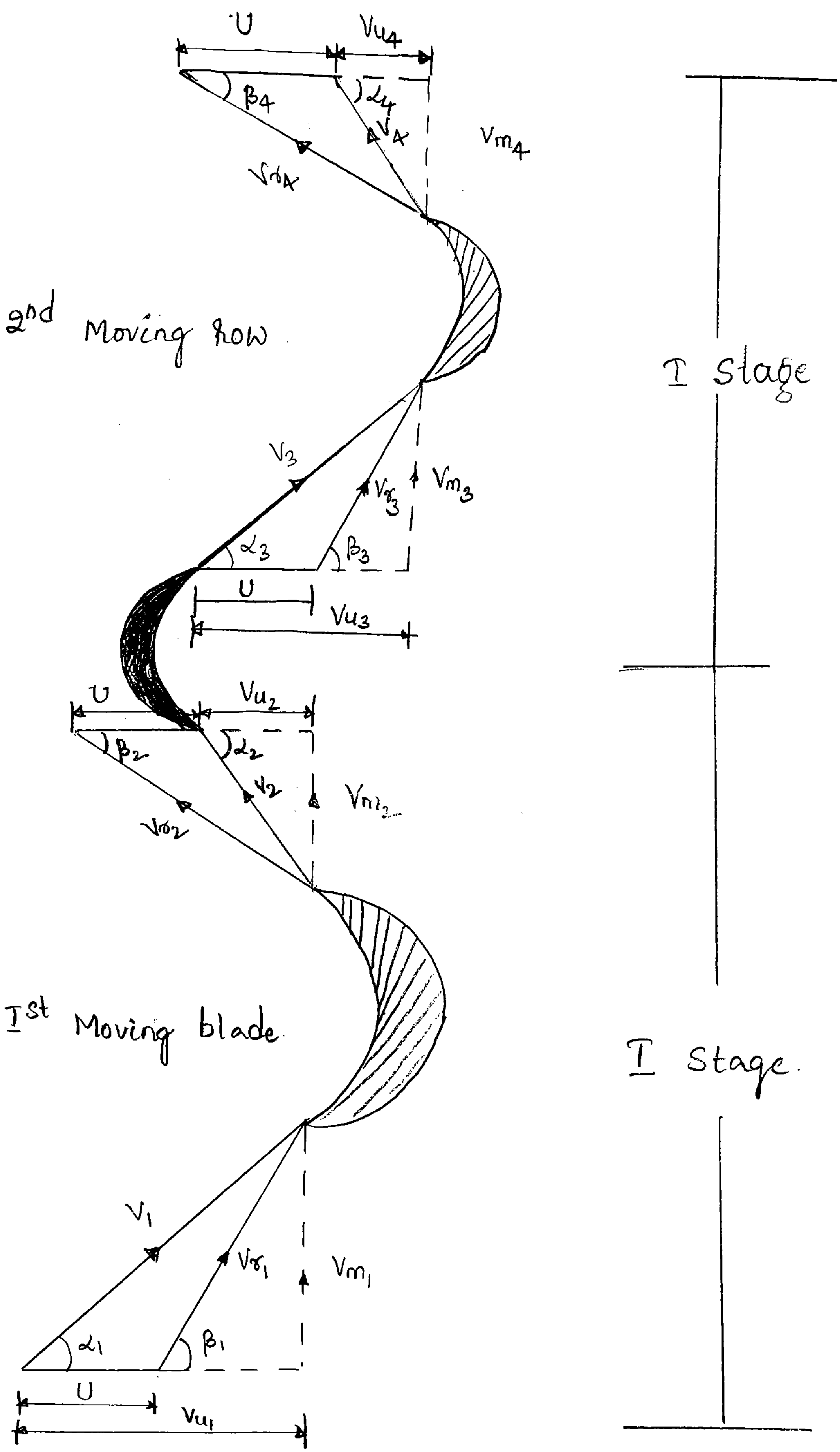
Specific power $\frac{P}{\dot{m}} = U[V_{u_1}+V_{u_2}] = 450\times 1050$

$$\frac{P}{\dot{m}} = \underline{\underline{472.5\ \text{kW}/(\text{kg/s})}}$$

Axial thrust F_a,

Since $V_{m_1}=V_{m_2}$, $\boxed{F_a = 0}$

Analysis of Two stages:



6. The following data refers to a velocity compounded Impulse steam turbine having two rows of moving blades and a fixed row between them. Velocity of steam leaving the nozzle is 1200m/s, nozzle angle is 20°, blade speed is 250m/s blade angle of first moving ~~e~~ row are equiangular, blade outlet angle of the fixed blade is 25°. Blade outlet angle of second moving row is 30°. Friction factor for all the rows is 0.9. Draw the velocity diagrams for a suitable scale and calculate the power developed, axial thrust, diagram efficiency for steam flow rate of 5000 kg/hr.

I-Stage		II-Stage	
Nozzle	Moving blade	Fixed blade	Moving blade
α_1	α_2	α_3	α_4
β_1	β_2	β_3	β_4
V_1	V_2	V_3	V_4
V_{r_1}	V_{r_2}	V_{r_3}	V_{r_4}

Given

Velocity compounded Impulse steam turbine

$V_1 = 1200\, m/s$, $\alpha_1 = 20°$

$U = 250\, m/s$, $\beta_1 = \beta_2$

blade outlet angle of fixed blade $\alpha_3 = 25°$.

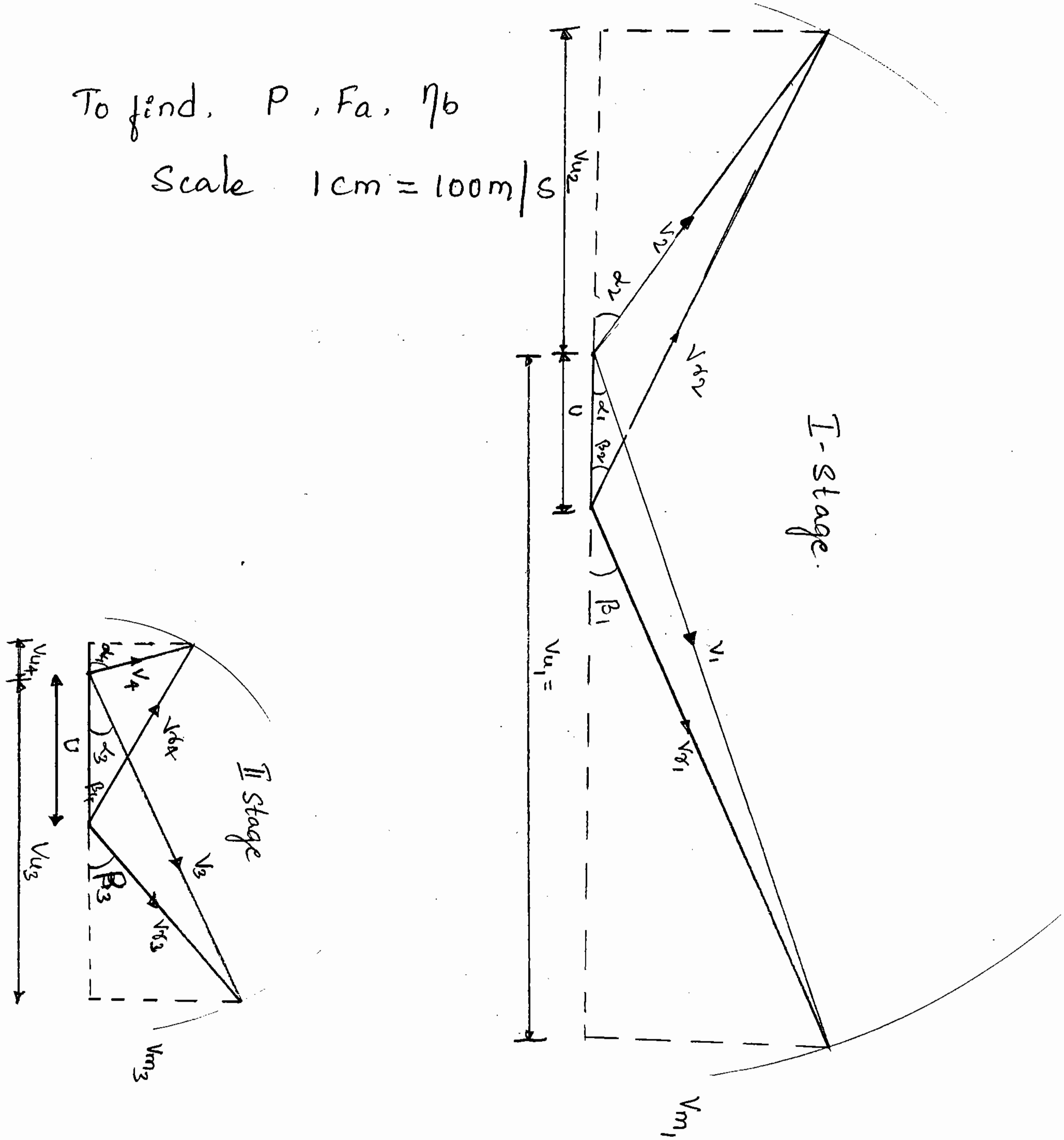
blade outlet angle of second moving blade $\beta_4 = 30°$

Friction factor $C_b = \frac{V_{r_2}}{V_{r_1}} = \frac{V_3}{V_2} = \frac{V_{r4}}{V_{r_3}} = 0.9$

mass flow rate of steam $\dot{m} = \frac{5000}{3600}$ kg/s

To find. P, Fa, η_b

Scale 1cm = 100m/s



From graph:

$V_{u_1} = 11.2\,cm = 1120\,m/s$

$V_{u_2} = 5.3\,cm = 530\,m/s$

$V_{m_1} = 4\,cm = 400\,m/s$

$V_{m_2} = 3.7\,cm = 370\,m/s$.

$V_{u_3} = 5.3\,cm = 530\,m/s$

$V_{u_4} = 0.5\,cm = 50\,m/s$.

$V_{m_3} = 2.5\,cm = 250\,m/s$

$V_{m_4} = 1.6\,cm = 160\,m/s$

we know

Power developed $P = \frac{\dot{m}}{g_c}\left[V_{u_1} \pm V_{u_2} + V_{u_3} \pm V_{u_4}\right]U$.

here $V_{u_1} \rightarrow$ $\quad$ $V_{u_3} \rightarrow$

$V_{u_2} \leftarrow$ $\quad$ $V_{u_4} \leftarrow$

$$P = \frac{5000}{3600}\left[1120 + 530 + 530 + 50\right]250$$

$$\boxed{P = 774.3\,kW}$$

also Axial thrust $F_a = \frac{\dot{m}}{g_c}\left[V_{m_1} - V_{m_2} + V_{m_3} - V_{m_4}\right]$

ii) $F_a = \frac{5000}{3600}[400 - 370 + 250 - 160]$

$$\boxed{F_a = 166.67\,N}$$

iii) Diagram efficiency $\eta_b = \frac{E}{(V_1^2/2)} = \frac{U[V_{u_1} + V_{u_2} + V_{u_3} + V_{u_4}]}{(V_1^2/2)}$

$$= \frac{250[1120 + 530 + 530 + 50]}{(1200^2/2)}$$

$$\boxed{\eta_b = 77.43\,\%}$$

7. In a curtis stage with two rotors, the steam velocity at the nozzle exit is 700 m/s. The outlet angles of the nozzle, the first rotor blade, the stator blade and the last rotor blade are respectively 17°, 23°, 19° and 37°. The blade velocity co-efficient is 0.93 for all the blades. If the mean blade speed is 160 m/s, when the steam flow rate is 2.7 kg/s. find

a) the power developed by the stage.

b) the stage efficiency if nozzle efficiency is 0.91

c) the axial thrust on the rotor.

d) the tangential forces acting on the blades.

I Stage		II Stage	
Nozzle	M.B	F.B	M.B
α_1	α_2	α_3	α_4
β_1	β_2	β_3	β_4
V_1	V_2	V_3	V_4
V_{r_1}	V_{r_2}	V_{r_3}	V_{r_4}

Given:

velocity of steam at nozzle exit ie $V_1 = 700\,m/s$

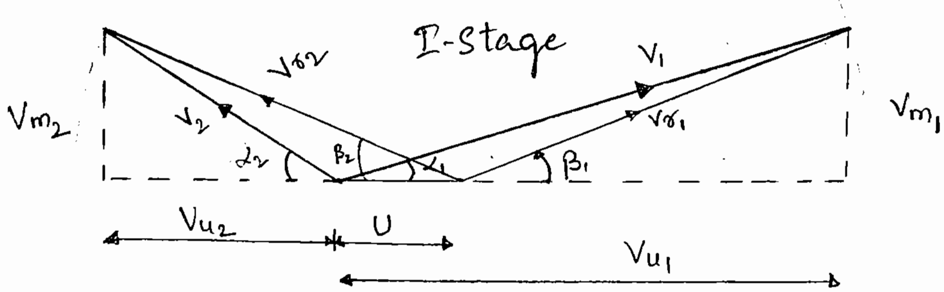
$\alpha_1 = 17^\circ$ $\qquad \alpha_3 = 19^\circ$

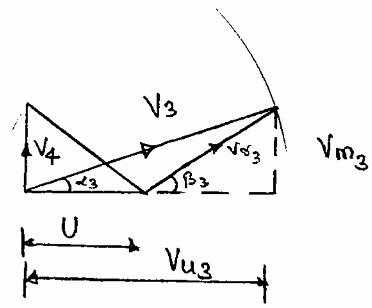
$\beta_2 = 23^\circ$ $\qquad \beta_4 = 37^\circ$

$$C_b = \frac{V_{r_2}}{V_{r_1}} = \frac{V_3}{V_2} = \frac{V_{r_4}}{V_{r_3}} = 0.93$$

$U = 160\,m/s$, $\dot{m} = 2.7\,kg/s$

Scale $1\,cm = 100\,m/s$





α_4 is nearly $90°$

From graph

$V_{r_1} = 5.5\,cm$ ie $550\,m/s$

but $C_b = \frac{V_{r_2}}{V_{r_1}}$ $\Rightarrow V_{r_2} = 0.93 \times 5.5 = 5.115\,cm$ or $511.5\,m/s$

again $V_{u_1} = 6.7\,cm = 670\,m/s$

$V_{u_2} = 3.1\,cm = 310\,m/s$

$V_{m_1} = 2\,cm = 200\,m/s$

$V_{m_2} = 1.9\,cm = 190\,m/s$.

again $V_2 = 3.7\,cm = 370\,m/s$

also $C_b = V_3/V_2$ $\Rightarrow V_3 = C_b V_2 = 0.93 \times 3.7 = 3.44\,cm$

$V_{u_3} = 3.3\,cm = 330\,m/s$

$V_{u_4} \simeq 0$, $V_{m_3} = 1.1\,cm = 110\,m/s$

$V_{m_4} = V_4 = 1.1\,cm = 110\,m/s$

We know $P = \dot{m} E = \dot{m} \left[V_{u_1} + V_{u_2} + V_{u_3} + V_{u_4} \right] U$

$$= 2.7 \times 160 \left[670 + 310 + 330 + 0 \right]$$

$$\boxed{P = 565.92 \text{ kW}}$$

ii) Stage efficiency $\eta_s = \eta_b \times \eta_n$.

$$\eta_b = \frac{E}{(V_1^2/2)} = \frac{160 \left[670 + 310 + 330 \right]}{(700^2/2)} = 0.8555$$

$$\eta_s = 0.8555 \times 0.91$$

$$\boxed{\eta_s = 77.85 \%}$$

iii) Axial thrust $F_a = \dot{m} \left[V_{m_1} \mp V_{m_2} + V_{m_3} - V_{m_4} \right]$

$$= 2.7 \left[200 - 190 + 110 - 110 \right]$$

$$\boxed{F_a = 27 \text{ N}}$$

iv) Tangential force $F_T = \dot{m} \left[V_{u_1} + V_{u_2} + V_{u_3} + V_{u_4} \right]$

$$= 2.7 \left[670 + 310 + 330 + 0 \right]$$

$$\boxed{F_T = 3537 \text{ N}}$$

8. In a two wheel Curtis-stage running with a mean rotor speed of 450 m/s, the steam leaves the second rotor axially. The nozzle angle is 16° and the rotor exit angles are $\beta_2 = 23^\circ$, $\beta_4 = 32^\circ$. The stator blade exit angle is 22°. If the blade velocity co-efficient is 0.941 in each blade, draw the velocity triangles and compute the rotor efficiency.

I-stage		II-stage	
Nozzle	Moving blade	Nozzle/Fixed blade	Moving blade
α_1	α_2	α_3	α_4
β_1	β_2	β_3	β_4
V_1	V_2	V_3	V_4
V_{r_1}	V_{r_2}	V_{r_3}	V_{r_4}

Given

Rotor speed $U = 450$ m/s

V_4 is axial. ie $\perp^{er}$

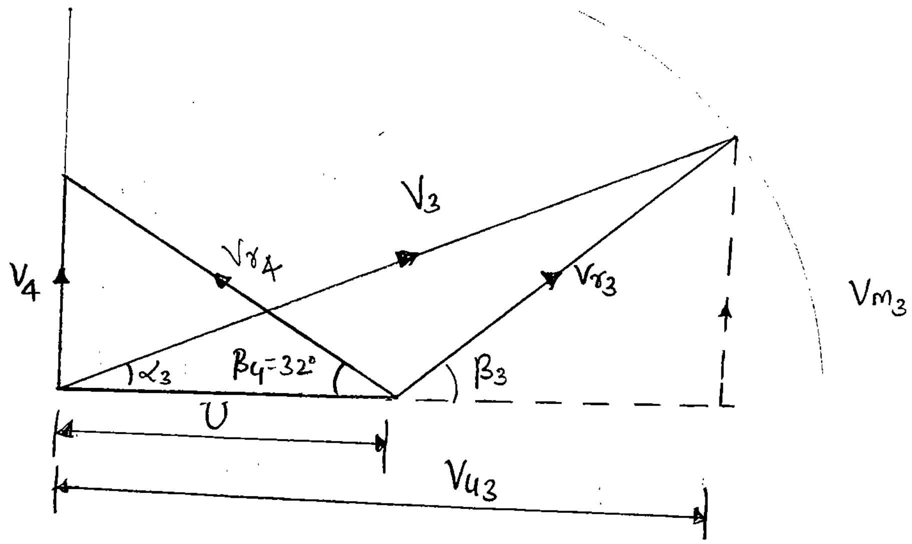
$\alpha_1 = 16^\circ$, $\beta_2 = 23^\circ$, $\beta_4 = 32^\circ$

$\alpha_3 = 22^\circ$, $C_b = 0.941$

To find η_r or η_b

procede from II-Stage

Scale $1 cm = 100 m/s$



From graph.

$V_{r4} = 5.2 cm$

ie $V_{r4} = 520 m/s$

also

$C_b = \dfrac{V_{r4}}{V_{r3}} = 0.91$ ie $V_{r3} = \dfrac{5.2}{0.91} = 5.7 cm$ or $570 m/s$

$V_{u3} = 8.8 cm = 8.8 \times 100 = \underline{\underline{880 m/s}}$.

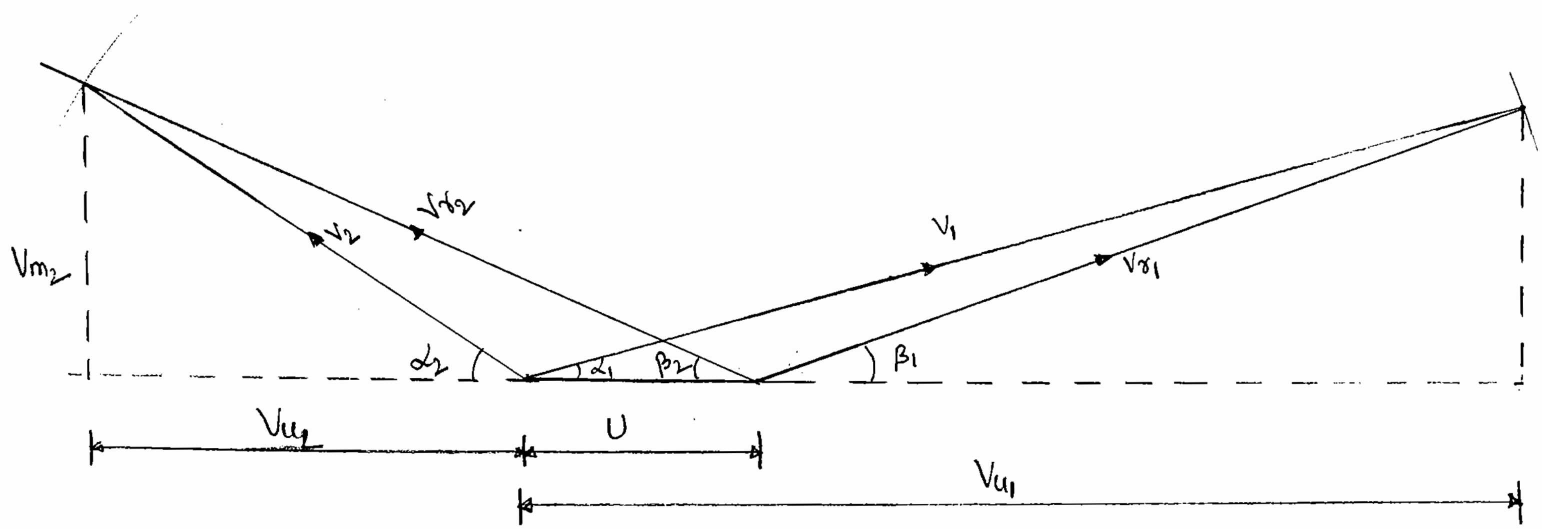
$V_{u4} = 0$

also $V_3 = 9.5 cm$ ie - $\underline{950 m/s}$

again $V_3/V_2 = 0.91$ or $V_2 = 10.44 cm$ y $1044 m/s$.

New Scale 1cm = 150 m/s

$U = 3\,cm$, $V_2 = 7\,cm$



From graph

$$V_{u_2} = 5.8\,cm = 5.8 \times 150 = 870\ m/s$$

$$V_{u_1} = 13.2\,cm = 13.2 \times 150 = 1980\ m/s$$

$$V_1 = 13.6\,cm = 13.6 \times 150 = 2040\ m/s$$

Rotor efficiency or blade efficiency

$$\eta_b = \frac{E}{V_1^2/2} = \frac{U[V_{u_1}+V_{u_2}+V_{u_3}+V_{u_4}]}{(V_1^2/2)}$$

$$= \frac{450[1980+870+880]}{(2040^2/2)}$$

$$\boxed{\eta_b = 80.67\ \%}$$

9. Steam enters the nozzles in a two-row Curtis wheel at 40 bar, 400°C. The pressure at the exit of the nozzle is 5 bar. If the speed-ratio U/V_1 is 0.2, the nozzle angle is 18° such that V_u is positive and the nozzle efficiency is 0.92 when the blade velocity co-efficient is 0.94, draw the velocity triangles and find a) power output if $m = 950\ kg/hr$.

b) Axial thrust, assume that blades are symmetric.

Given

Two row curtis wheel turbine

$P_1 = 40\,atm = 40\,bar$, $T_1 = 400^\circ C$

$P_2 = 5\,atm = 5\,bar$

$\frac{U}{V_1} = 0.2$, $\alpha_1 = 18^\circ$ $\eta_n = 0.92$

$C_b = \frac{V_{r_2}}{V_{r_1}} = \frac{V_3}{V_2} = \frac{V_{r4}}{V_{r_3}} = 0.94$

$\dot{m} = 950\,kg/hr$, blades are symmetric. $\beta_1 = \beta_2 = \beta_3 = \beta_4$

$P = ?$, $F_a = ?$

From Mollier diagram

$h_1 = 3210\,kJ/kg$ at $P_1 = 40\,bar, T = 400^\circ C$

$h_2 = 2750\,kJ/kg$ at $P_2 = 5\,bar$

$$\eta_{nozzle} = \frac{(V_1^2/2)}{h_1 - h_2'}$$

$$0.92 = \frac{V_1^2/2}{(3210-2750)\times 10^3}$$

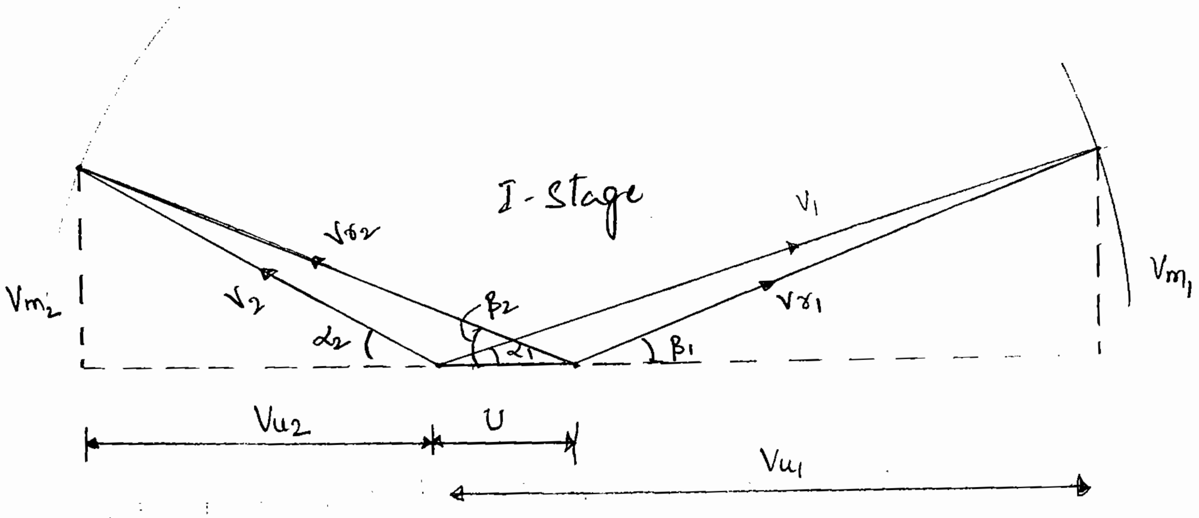
$$\boxed{V_1 = 920\,m/s}$$

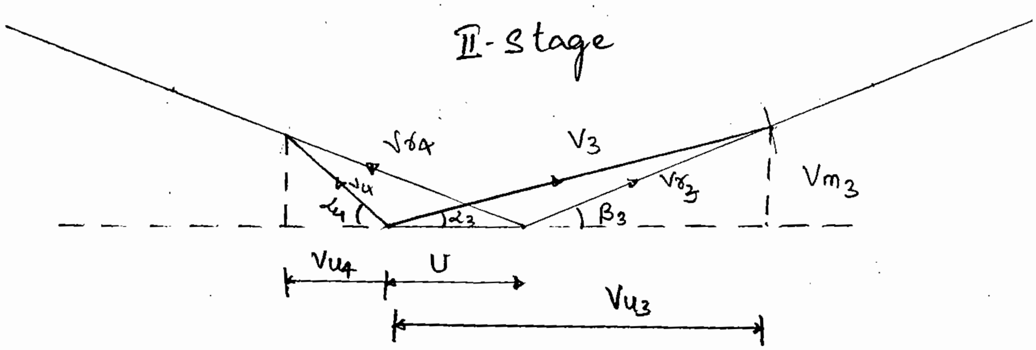
again $\frac{U}{V_1} = 0.2$

$U = 0.2 \times 920 = 184\,m/s$

$$\boxed{U = 184\,m/s}$$

Scale 1cm = 100 m/s





From graph

$V_{r_1} = 7.5$ cm ie 750 m/s

also $C_b = \frac{V_{r2}}{V_{r_1}} = 0.94 \Rightarrow V_{r_2} = 0.94 \times 7.5 = 7.05$ cm

$V_2 = 5.5$ cm ie $\underline{V_2 = 550 \text{ m/s}}$

also $C_b = V_3/V_2$

$V_3 = 0.94 \times 5.5 = 5.17$ cm $\simeq$ 5.2 cm

ie $\boxed{V_3 = 520 \text{ m/s}}$

$V_{r_3} = 3.5\,cm \Rightarrow V_{r_3} = \underline{350\,m/s}$

$C_b = \frac{V_{r4}}{V_{r3}}$

$V_{r4} = 0.94 \times 3.5 = 3.29\,cm \Rightarrow 3.3\,cm$

$\underline{\underline{V_{r4} = 330\,m/s}}$

From graph

$V_{u1} = 8.8\,cm = 880\,m/s$

$V_{u2} = 4.7\,cm = 470\,m/s$.

$V_{m1} = 2.8\,cm = 280\,m/s$

$V_{m2} = 2.6\,cm = 260\,m/s$

$V_{u3} = 5\,cm = 500\,m/s$

$V_{u4} = 1.4\,cm = 140\,m/s$

$V_{m3} = 1.3\,cm = 130\,m/s$

$V_{m4} = 1.2\,cm = 120\,m/s$

we know.

Power $P = \dot{m}E = \frac{950}{3600} \times 184 [880 + 470 + 500 + 140]$

$\underline{\underline{P = 96.625\,kW}}$

Axial thrust $F_a = \dot{m}\left[V_{m_1} - V_{m_2} + V_{m_3} - V_{m_4}\right]$

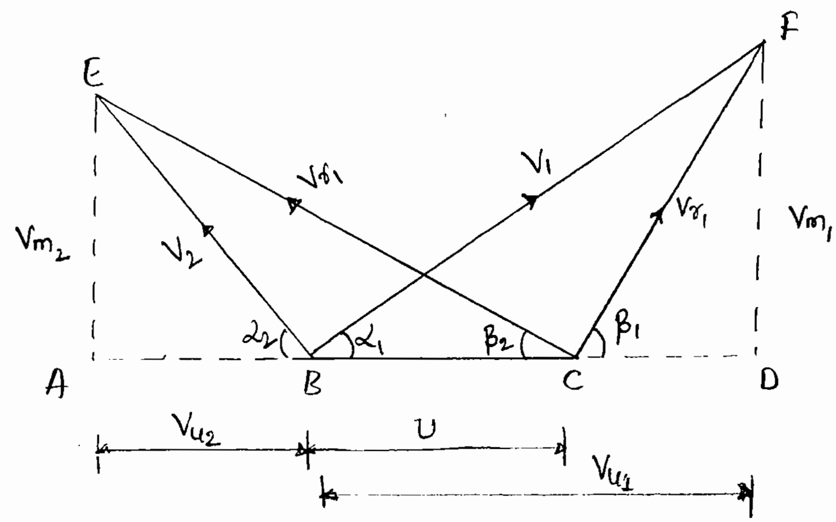
$$= \frac{950}{3600}\left[280 - 260 + 130 - 120\right]$$

$$\boxed{F_a = 7.92\,N}$$

Prove that the maximum ~~stage~~ rotor efficiency of parson's (50% reaction) turbine is given by $\eta_{b\,max} = \dfrac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$

OR

Stage efficiency $\eta_{s\,max} = \dfrac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$ ie $\eta_n = 1$



We know,

Rotor & Blade effeciency $= \dfrac{\text{Utilized energy}}{\text{Total energy supplied.}}$

ie

$$\eta_b = \frac{E}{\frac{V_1^2}{2} + \left(\frac{V_{r_2}^2 - V_{r_1}^2}{2}\right)} \qquad \text{—— (a)}$$

also

$$\eta_b = \frac{U[V_{u_1} + V_{u_2}]}{\frac{V_1^2}{2} + \left(\frac{V_{r_2}^2 - V_{r_1}^2}{2}\right)} \qquad \text{—— (1)}$$

consider $V_{u_1} + V_{u_2} = AB + BD$ (from vel. triangle)

also $\cos\alpha_1 = \frac{V_{u_1}}{V_1}$ or $V_{u_1} = BD = V_1 \cos\alpha_1$

$\cos\alpha_2 = \frac{V_{u_2}}{V_2}$ or $V_{u_2} = AB = V_2 \cos\alpha_2$.

ie $V_{u_1} + V_{u_2} = V_1 \cos\alpha_1 + V_2 \cos\alpha_2$.

$= V_1 \cos\alpha_1 + AC - BC$

$= V_1 \cos\alpha_1 + V_{r_2} \cos\beta_2 - U$ from velocity triangle

but for 50% reaction

$V_{r_1} = V_2, \quad V_1 = V_{r_2}$.

$\Rightarrow \quad V_{u_1} + V_{u_2} = V_1 \cos\alpha_1 + V_1 \cos\alpha_1 - U$

$$V_{u_1} + V_{u_2} = 2V_1 \cos\alpha_1 - U \qquad \text{—— (2)}$$

consider

$$\frac{V_1^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2} = \frac{V_1^2}{2} + \frac{V_{r_2}^2}{2} - \frac{V_{r_1}^2}{2}.$$

$$= \frac{V_1^2}{2} + \frac{V_1^2}{2} - \frac{V_{r_1}^2}{2} \qquad \because V_{r_2} = V_1$$

$$= V_1^2 - \frac{V_{r_1}^2}{2} \qquad \text{—— (3)}$$

$$V_{r_1}^2 = V_{m_1}^2 + (V_{u_1} - U)^2$$

$$= V_{m_1}^2 + V_{u_1}^2 + U^2 - 2UV_{u_1}$$

also $V_{m_1}^2 = V_1^2 - V_{u_1}^2$

$$\Rightarrow \quad V_{r_1}^2 = V_1^2 - V_{u_1}^2 + V_{u_1}^2 + U^2 - 2UV_{u_1}$$

$$= V_1^2 + U^2 - 2UV_{u_1}$$

$$V_{r_1}^2 = V_1^2 + U^2 - 2UV_1\cos\alpha_1 \quad \text{— (4)} \qquad \therefore \cos\alpha_1 = \frac{V_{u_1}}{V_1}$$

Substitute (4) in (3)

$$\frac{V_1^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2} = V_1^2 - \left(\frac{V_1^2 + U^2 - 2UV_1\cos\alpha_1}{2}\right)$$

$$= \frac{V_1^2 - U^2 + 2UV_1\cos\alpha_1}{2} \quad \text{— (5)}.$$

Substitute (5) and (2) in (1)

(1) $\Rightarrow$

$$\eta_b = \frac{U\left[2V_1\cos\alpha_1 - U\right]}{\left(\frac{V_1^2 - U^2 + 2UV_1\cos\alpha_1}{2}\right)}$$

$$= \frac{UV_1\left(2\cos\alpha_1 - \frac{U}{V_1}\right)}{\frac{V_1^2}{2}\left(1 - \frac{U^2}{V_1^2} + 2\frac{U}{V_1}\cos\alpha_1\right)}$$

$$= \frac{\frac{2U}{V_1}\left[2\cos\alpha_1 - \frac{U}{V_1}\right]}{1 - \frac{U^2}{V_1^2} + 2\frac{U}{V_1}\cos\alpha_1}$$

but $\frac{U}{V_1}$ = Speed ratio ϕ

$$\eta_b = \frac{2\phi(2\cos\alpha_1 - \phi)}{1-\phi^2+2\phi\cos\alpha_1} = \frac{4\phi\cos\alpha_1 - 2\phi^2}{1-\phi^2+2\phi\cos\alpha_1} \quad \text{--- (b)}$$

For maximum efficiency.

$$\frac{\partial \eta_b}{\partial \phi} = 0 .$$

ie $$\frac{\partial}{\partial \phi}\left[\frac{4\phi\cos\alpha_1 - 2\phi^2}{1-\phi^2+2\phi\cos\alpha_1}\right] = 0$$

$$\frac{(1-\phi^2+2\phi\cos\alpha_1)(4\cos\alpha_1 - 4\phi) - (4\phi\cos\alpha_1 - 2\phi^2)(-2\phi+2\cos\alpha_1)}{(1-\phi^2+2\phi\cos\alpha_1)^2} = 0 .$$

$$(1-\phi^2+2\phi\cos\alpha_1)(4\cos\alpha_1 - 4\phi) - (4\phi\cos\alpha_1 - 2\phi^2)(-2\phi+2\cos\alpha_1) = 0$$

$$(-2\phi+2\cos\alpha_1)\left[2(1-\phi^2+2\phi\cos\alpha_1) - (4\phi\cos\alpha_1 - 2\phi^2)\right] = 0$$

$$-2\phi + 2\cos\alpha_1 = 0 \quad \text{or} \quad 2(1-\phi^2+2\phi\cos\alpha_1) = 4\phi\cos\alpha_1 - 2\phi^2 .$$

$$\Rightarrow \quad \boxed{\phi_{opt} = \cos\alpha_1}$$

or $$\left(\frac{U}{V_1}\right)_{opt} = \cos\alpha_1 \quad \text{--- (6)}$$

Sub (6) in η_b (b)

$$\eta_{b\,max} = \frac{4\cos\alpha_1 \cos\alpha_1 - 2\cos^2\alpha_1}{1-\cos^2\alpha_1 + 2\cos^2\alpha_1 \cdot \cos\alpha_1}$$

$$\boxed{\eta_{b\,max} = \frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}}$$

or in terms of speed ratio.

$$\eta_{b\,max} = \frac{2\phi^2_{opt}}{1+\phi^2_{opt}}$$

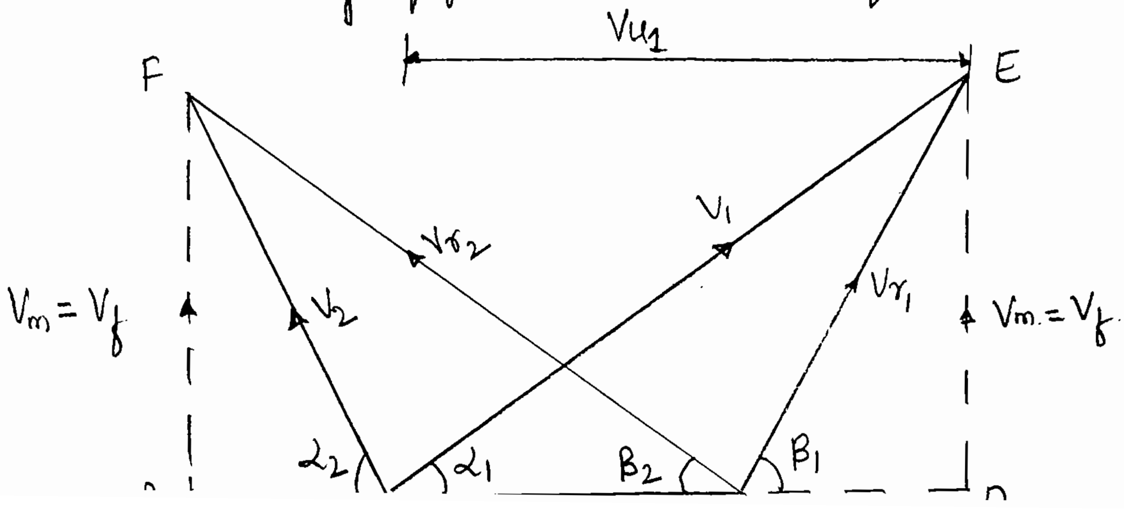
Prove that degree of reaction for an axial flow device (Turbine) (assuming constant velocity of flow) is given by

$$R = \frac{V_f}{2U}\left(\frac{\tan\beta_1 - \tan\beta_2}{\tan\beta_1 \tan\beta_2}\right)$$

Given.

Axial flow device $U_1 = U_2 = U$

velocity of flow $V_{m_1} = V_{m_2} = V_m = V_f$ (V_f & V_m are same)



We know, Degree of reaction R is

$$R = \frac{\text{Change in static head}}{\text{change in total head.}}$$

$$= \frac{\frac{1}{2}(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{\frac{1}{2}(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}$$

but $U_1 = U_2$.

$$\Rightarrow \quad R = \frac{(V_{r_2}^2 - V_{r_1}^2)/2}{E} = \frac{(V_{r_2}^2 - V_{r_1}^2)/2}{U[V_{u_1} + V_{u_2}]}$$

$$R = \frac{(V_{r_2}^2 - V_{r_1}^2)}{2U[V_{u_1} + V_{u_2}]} \quad \text{— (1)}$$

From velocity triangle.

$$\sin\beta_2 = \frac{V_{m_2}}{V_{r_2}}, \qquad \sin\beta_1 = \frac{V_{m_1}}{V_{r_1}}$$

$$V_{r_2} = V_{m_2} \operatorname{cosec}\beta_2 \text{ — (a)}. \qquad V_{r_1} = V_{m_1} \operatorname{cosec}\beta_1 \text{ — (b)}$$

$$\tan\beta_2 = \frac{V_{m_2}}{AC}, \qquad \tan\beta_1 = \frac{V_{m_1}}{CD}.$$

$$AC = V_{m_2}/\tan\beta_2 = V_{m_2}\cot\beta_2 \text{ — (c)}, \qquad CD = V_{m_1}\cot\beta_1 \text{ — (d)}.$$

but $V_{u_1} + V_{u_2} = AD = AC + CD$.

hence. $V_{u_1} + V_{u_2} = V_{m_2}\cot\beta_2 + V_{m_1}\cot\beta_1$ — (e)

Put a, b and e in (1)

① $\Rightarrow$

$$R = \frac{\left(V_m^2 \operatorname{cosec}^2\beta_2 - V_m^2 \operatorname{cosec}^2\beta_1\right)}{2U\left[V_m \cot\beta_2 + V_m \cot\beta_1\right]}$$

Since V_m is constant

$$R = \frac{V_m^2}{2UV_m}\,\frac{\left[\operatorname{cosec}^2\beta_2 - \operatorname{cosec}^2\beta_1\right]}{\left[\cot\beta_2 + \cot\beta_1\right]}$$

$$= \frac{V_m}{2U}\,\frac{\left[(1+\cot^2\beta_2) - (1+\cot^2\beta_1)\right]}{(\cot\beta_2 + \cot\beta_1)}$$

$$= \frac{V_m}{2U}\left[\frac{\cot^2\beta_2 - \cot^2\beta_1}{\cot\beta_2 + \cot\beta_1}\right] = \frac{(\cot\beta_2 + \cot\beta_1)(\cot\beta_2 - \cot\beta_1)}{(\cot\beta_2 + \cot\beta_1)}$$

$$\boxed{R = \frac{V_m}{2U}\left(\cot\beta_2 - \cot\beta_1\right) = \frac{V_f}{2U}\left(\cot\beta_2 - \cot\beta_1\right)}$$

$$R = \frac{V_m}{2U}\left(\frac{1}{\tan\beta_2} - \frac{1}{\tan\beta_1}\right) = \frac{V_m}{2U}\left[\frac{\tan\beta_1 - \tan\beta_2}{\tan\beta_1 \tan\beta_2}\right]$$

ie

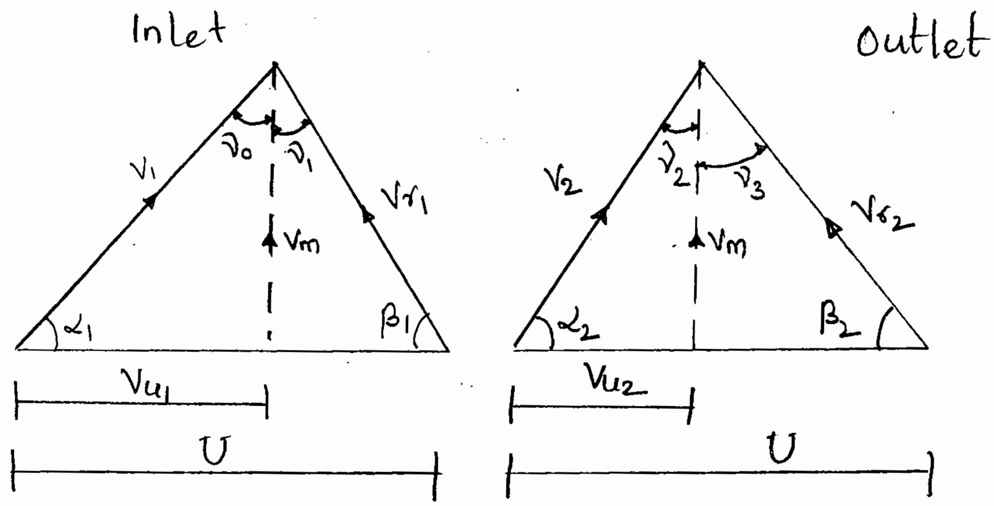
$$\boxed{R = \frac{V_m}{2U}\left(\frac{\tan\beta_1 - \tan\beta_2}{\tan\beta_1 \tan\beta_2}\right) = \frac{V_f}{2U}\left(\frac{\tan\beta_1 - \tan\beta_2}{\tan\beta_1 \tan\beta_2}\right)}$$

This is for axial flow turbines

* Prove that $R = \frac{V_m}{2U}\left[\frac{\tan\beta_2 + \tan\beta_1}{\tan\beta_1 \cdot \tan\beta_2}\right]$

for Axial flow compressors, Blowers, Pumps.

velocity triangle for Axial flow power absorbing devices.



Energy transfer, $E = U[V_{u_1} \pm V_{u_2}]$

$$E = U[V_{u_1} - V_{u_2}]$$

Since $V_{u_1} \rightarrow$
$V_{u_2} \rightarrow$

Here E is negative $\therefore$ Energy absorbed.

To make positive

$$E = U[V_{u_2} - V_{u_1}] \quad \text{--- (1)}$$

From inlet velocity triangle.

$$\tan\theta_1 = \frac{U - V_{u1}}{V_m}$$

$$\Rightarrow \quad V_{u_1} = U - V_m \tan\theta_1 \quad \text{--- (a)}$$

From outlet velocity triangle

$$\tan \vartheta_3 = \frac{U - V_{u_2}}{V_m}$$

$$V_{u_2} = U - V_m \tan \vartheta_3 \quad \text{—— (b)}$$

Substitute (a) and (b) in (1)

(1) $\Rightarrow$

$$E = U\left[U - V_m \tan \vartheta_3 - U + V_m \tan \vartheta_1\right]$$

$$E = U V_m\left[\tan \vartheta_1 - \tan \vartheta_3\right] \quad \text{—— (c)}$$

but $\vartheta_1 = 90 - \beta_1$, $\tan \vartheta_1 = \tan(90 - \beta_1) = \cot \beta_1$

$\vartheta_3 = 90 - \beta_2$ $\quad \tan \vartheta_3 = \tan(90 - \beta_2) = \cot \beta_2$.

$$\Rightarrow \quad E = U V_m\left[\cot \beta_1 - \cot \beta_2\right] \quad \text{—— (2)}.$$

We know Degree of Reaction

$$R = \frac{(V_{r_2}^2 - V_{r_1}^2)/2}{E}$$

Since here E is negative

$$R = \frac{(V_{r_1}^2 - V_{r_2}^2)}{2E} \quad \text{—— (3)}$$

from inlet and outlet vel. triangles.

$$\cos\vartheta_1 = \frac{V_m}{V_{r_1}} \qquad \cos\vartheta_3 = \frac{V_m}{V_{r_2}}$$

$$V_{r_1}^2 = V_m^2 \sec^2\vartheta_1 \quad \text{— (d)} \qquad V_{r_2}^2 = V_m^2 \sec^2\vartheta_3 \quad \text{— (e)}.$$

put (d), (e) and (c) in (3).

(3) $\Rightarrow$

$$R = \frac{V_m^2 \sec^2\vartheta_1 - V_m^2 \sec^2\vartheta_3}{2UV_m(\tan\vartheta_1 - \tan\vartheta_3)} = \frac{V_m}{2U}\frac{\left[1+\tan^2\vartheta_1 - (1+\tan^2\vartheta_3)\right]}{\tan\vartheta_1 - \tan\vartheta_3}.$$

$$= \frac{V_m}{2U}\frac{(\tan\vartheta_1 + \tan\vartheta_3)(\tan\vartheta_1 - \tan\vartheta_3)}{(\tan\vartheta_1 - \tan\vartheta_3)}$$

$$R = \frac{V_m}{2U}(\tan\vartheta_1 + \tan\vartheta_3)$$

$$R = \frac{V_m}{2U}\left[\cot\beta_1 + \cot\beta_2\right] = \frac{V_m}{2U}\left[\frac{1}{\tan\beta_1} + \frac{1}{\tan\beta_2}\right]$$

$$R = \frac{V_m}{2U}\left(\frac{\tan\beta_2 + \tan\beta_1}{\tan\beta_1 + \tan\beta_2}\right)$$

HYDRAULIC TURBINES

Hydraulic or water turbines are the machines which convert the water energy (hydro power) into mechanical energy. The water energy may be either in the form of potential energy as we find in dams, reservoirs or in the form of kinetic energy in flowing water.

Classification

Hydraulic turbines may be classified as follows.

1. Based on the type of energy at inlet to the turbine

   a) Impulse turbine:

   The energy is in the kinetic form. Example: pelton wheel, Turgo wheel.

   b) Reaction turbine:

   The energy is in ~~th~~ both kinetic and pressure form. Example: Francis turbine, Kaplan turbine.

2. Based on the direction of flow of water through the runner.

a) Tangential flow or peripheral flow

Water flows in a direction tangential to the path of rotation ie perpendicular to both axial and radial directions. Example: Pelton wheel.

b) Radial inward or outward flow

In radial flow machine, the water flows along the radial direction and flow remains normal to the axis of rotation as it passes through the runner. It may be inward flow or outward flow.

In Inward flow turbines, the water enters at the outer periphery and passes through the runner inwardly towards the axis of rotation and finally leaves at inner periphery.
Example: Francis turbine, Thomson turbine, ...

In outward flow turbines, the water enters at the inner periphery and leaves at outer periphery.
Example: Forneyron turbine.

c) Axial flow:

Water flows parallel to the axis of the turbine
Example: Girard, Jonval, Kaplan turbine.

d) Mixed flow or Diagonal flow

In this type of turbine, the flow of fluid enters radially and leaves the runner axially or enters the runner axially and leaves radially

Example: Modern Francis turbine, Deriaz turbine.

3. Based on the head under which turbine works.

a) High head turbine ex: Pelton wheel.

b) Medium head turbine Ex: Francis turbine

c) Low head turbine Ex: Kaplan turbine

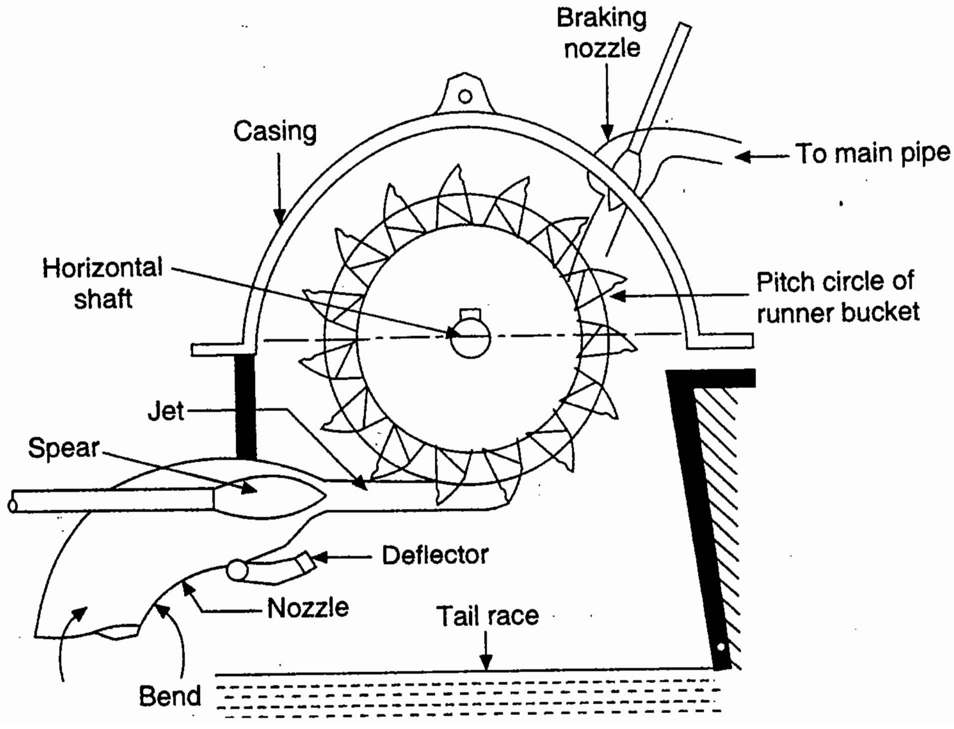
4. Based on the Specific Speed of the turbine.

a) High specific speed ex: Kaplan turbine

b) Medium Specific Speed ex: Francis turbine.

c) Low Specific Speed ex: Pelton wheel.

Pelton wheel



It is an impulse turbine working under a high head and handling low quantity of water. The specific speed is in the range of 8.5 to 51 rpm.

The water flows from the reservoir to the turbine through the penstock. The end of the penstock is fitted with one or more nozzles. The entire pressure energy of water is converted into kinetic energy in the nozzle. The high velocity water jet emerging from the nozzle strikes the bucket attached to the periphery of the rotor and sets the bucket into rotary motion. Here, water flows in the tangential direction, doing work. The kinetic energy of the jet is completely transferred to the rotating wheel, ie the velocity of water at the exit of the runner is just sufficient to enable it to move out the runner. The static pressure of water at the entrance and exit of the bucket is same.

Terminology.

Gross Head (H_g)

It is the head of water available above the centre line of the jet for doing useful work.

Pipe losses (h_f)

Some amount of head is lost in pipe fittings (bends, elbows, etc..) and friction in the pipe.

$$h_f = \frac{4fLV^2}{2gD}$$

Effective head (H)

It is the head of water available at the inlet of the nozzle.

ie $\quad H = H_g - h_f$

Turbine Efficiency.

Following are the important efficiencies of a hydraulic turbine.

1) Hydraulic efficiency.

It is the ratio of power developed by the runner to the water power available at the inlet of the turbine.

$$\text{ie} \quad \eta_H = \frac{\text{Power dev. by runner}}{\text{Power of water}} = \frac{\dot{m}(U_1 V_{u_1} \pm U_2 V_{u_2})}{\rho g Q H}$$

2. Mechanical Efficiency (η_m)

It is the ratio of the ~~quality~~ ~~quan~~ shaft power output by the turbine to the power developed by the runner.

$$\eta_{mech} = \frac{SP}{\dot{m}u[V_{u_1} \pm V_{u_2}]}$$

3. Volumetric Efficiency (η_v)

It is the ratio of the quantity of water actually striking the runner to the quantity of water supplied to the runner.

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water leaving the nozzle.}}$$

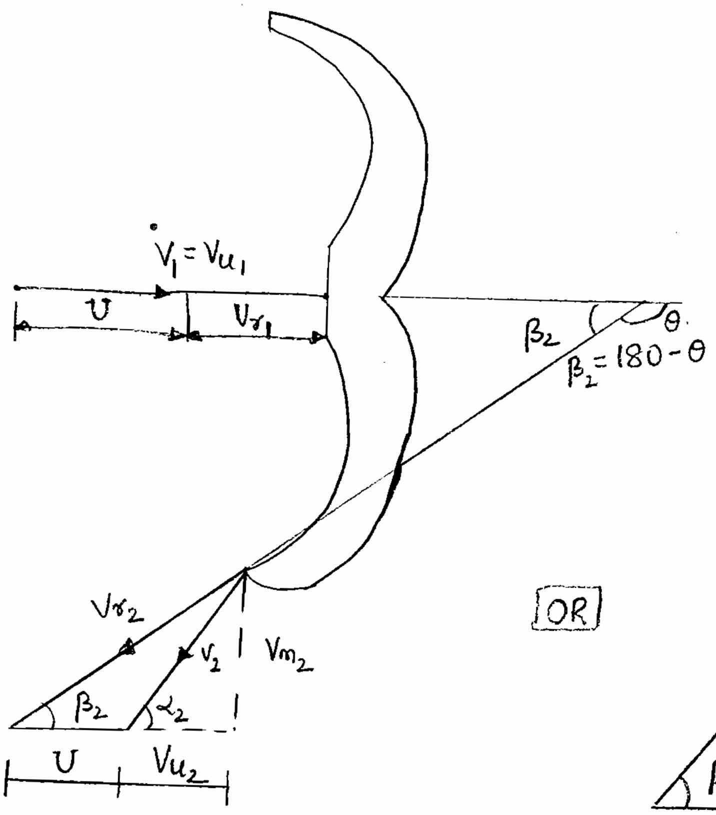
4. Overall efficiency (η_o)

It is the ratio of shaft output power by the turbine to the water power available at inlet of the turbine.

$$\eta_o = \frac{SP}{\rho g Q H}$$

also $\eta_o = \eta_H \, \eta_{vol} \cdot \eta_{mech}.$

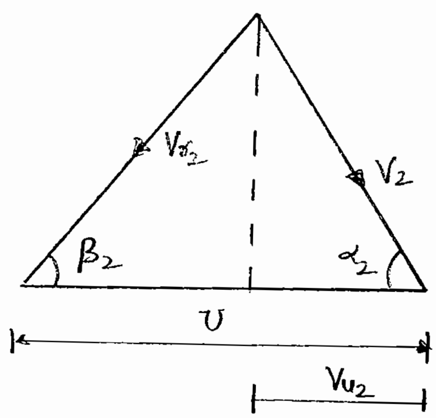
Velocity triangles for pelton wheel



when, $V_{u_1} \rightarrow$

$V_{u_2} \leftarrow$

OR



$V_{u_1} \rightarrow$,

$V_{u_2} \rightarrow$

Work done and condition for maximum efficiency.

Above triangles shows the inlet and outlet velocity triangles. Since the angle of entrance of jet is zero, the inlet velocity triangle collapses to a straight line. The tangential component of absolute velocity at inlet $V_{u_1} = V_1$ and relative velocity at the inlet is $V_{r_1} = V_1 - U$.

From outlet velocity triangle,

$$V_{u_2} = V_{r_2}\cos\beta_2 - U \qquad \therefore \cos\beta_2 = \frac{U+V_{u_2}}{V_{r_2}}.$$

assume there is no losses due to friction

ie $V_{r_1} = V_{r_2}$.

$$\Rightarrow \quad V_{u_2} = V_{r_1}\cos\beta_2 - U$$

$$V_{u_2} = (V_1 - U)\cos\beta_2 - U \quad \text{—— (1)}$$

We have from Euler equation,

Workdone / kg of water by the runner

$$W = \frac{U[V_{u_1} \pm V_{u_2}]}{g_c}$$

From velocity triangles V_{u_1} and V_{u_2} are opposite

$$\Rightarrow \quad W = \frac{U}{g_c}[V_{u_1} + V_{u_2}]$$

$$= \frac{U}{g_c}\left[V_{u_1} + (V_1 - U)\cos\beta_2 - U\right]$$

$$= \frac{U}{g_c}\left[(V_1 - U) + (V_1 - U)\cos\beta_2\right] \qquad \therefore V_1 = V_{u_1}$$

$$W = \frac{(V_1 - U)U}{g_c}(1 + \cos\beta_2)$$

if bucket velocity co-efficient $C_b = \dfrac{V_{r_2}}{V_{r_1}}$ is considered then,

$$W = \frac{U(V_1 - U)}{g_c}(1 + C_b\cos\beta_2) \quad \text{—— (2)}$$

and energy supplied to the wheel is in the form of kinetic energy of the jet which is equal to $\dfrac{V_1^2}{2g_c}$ —— (3)

1. A pelton wheel produces power of 23000kW under a head of 1770m while running at 750rpm. Estimate from the turbine jet diameter, mean diameter of runner, number of jets and number of buckets. Assume co-efficient of velocity as 0.97, turbine efficiency 0.85 and speed ratio 0.46.

Given

$P = 23000\,kW$, $N = 750\,rpm$

$H = 1770\,m$, $\eta_t = 0.85$, $\phi = \frac{U}{V_1} = 0.46$

To find d, D, n, z

We know,

Tangential speed $U = \phi\sqrt{2gH}$

$U = 0.46\sqrt{2 \times 9.81 \times 1770}$

$U = \underline{85.72}\ m/s$

also Absolute velocity $V_1 = C_v\sqrt{2gH}$

$V_1 = 0.97\sqrt{2 \times 9.81 \times 1770}$

$V_1 = \underline{180.76}\ m/s$

we have

$U = \frac{\pi D N}{60} \Rightarrow 85.72 = \frac{\pi D \times 750}{60}$

$\boxed{D = 2.183\,m}$

power developed $P = \eta \frac{\rho g Q H}{1000}$

$$23 \times 10^3 = 0.85 \times \frac{1000 \times 9.81 \times Q \times 1770}{1000}$$

Total discharge, $Q = 1.55836 \, m^3/s$

but discharge through each nozzle

$$Q = n.q$$

$$1.55836 = n.q \quad \text{—} (1)$$

we know Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}}$ — p in kw

$$= \frac{750\sqrt{23000}}{1770^{5/4}}$$

$$N_s = 9.907 \leq 35$$

$\Rightarrow$ Single jet ie $n = 1$

(1) $\Rightarrow$

$$Q = 1.q = 1.55836 \, m^3/s. = \frac{\pi}{4} d^2 V_1$$

$$1.55836 = \frac{\pi}{4} \times d^2 \times 180.76$$

$$\boxed{d = 0.105 \, m}$$

Jet ratio $m = \frac{D}{d}$

$$m = \frac{2.183}{0.105} = \underline{\underline{20.76}}$$

Number of buckets $Z = \frac{D}{2d} + 15$

$$= \frac{2.183}{2 \times 0.105} + 15 = 25.4$$

$Z \simeq 26$

width of buckets $W = 2.8d$ to $3.2d$

$W = 3d$ (say)

$W = 3 \times 0.105$

$W = \underline{\underline{0.315 m}}$

2. At certain stage, Pelton wheel produces 31400 HP under a head of 1750m running at 700rpm. Estimate for the turbine a) number of jets b) jet diameter c) mean diameter of runner d) number of buckets. Assume velocity co-efficient is 0.86, bucket angle at exit is 152°. Draw inlet and outlet velocity triangles and find tangential force, the mass flow rate is 5 kg/s. Assume $C_v = 0.97$, $\phi = 0.46$.

Given

$P = 31400 \text{ HP} = 23414.98 \text{ kW}$

$H = 1750 m \quad N = 700 rpm$

$C_v = 0.86, \quad C_v = 0.97, \quad \phi = 0.46$

To find: n, d, D, Z, F_T

we have

absolute velocity $V_1 = C_v\sqrt{2gH}$

$$V_1 = 0.97\sqrt{2\times 9.81\times 1750}$$

$$\underline{V_1 = 179.74\, m/s}$$

Tangential blade speed $U = \phi\sqrt{2gH}$

$$U = 0.46\sqrt{2\times 9.81\times 1750}$$

$$\underline{U = 85.24\, m/s}$$

Diameter of the runner D,

$$U = \frac{\pi DN}{60}$$

$$85.24 = \frac{\pi D\times 700}{60}$$

$$D = 2.3257$$

$$\boxed{D \simeq 2.33\, m}$$

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

$$= \frac{700\sqrt{23414.98}}{1750^{5/4}}$$

$$N_s = 9.46 \leq 35$$

$$\Rightarrow \underline{\underline{n=1}}\ \text{(Single jet)}$$

Power P,

$$P = \eta\,\frac{\rho g Q H}{1000}$$

$$23414.98 = \frac{0.85\times 9.81\times 1000\times Q\times 1750}{1000}$$

$Q = 1.6046\ m^3/s$

we know, $Q = n.q$

$Q = q$ for $n = 1$

$\Rightarrow \frac{\pi}{4} d^2 v_1 = q$

$\frac{\pi}{4} d^2 \times 179.74 = 1.6046$

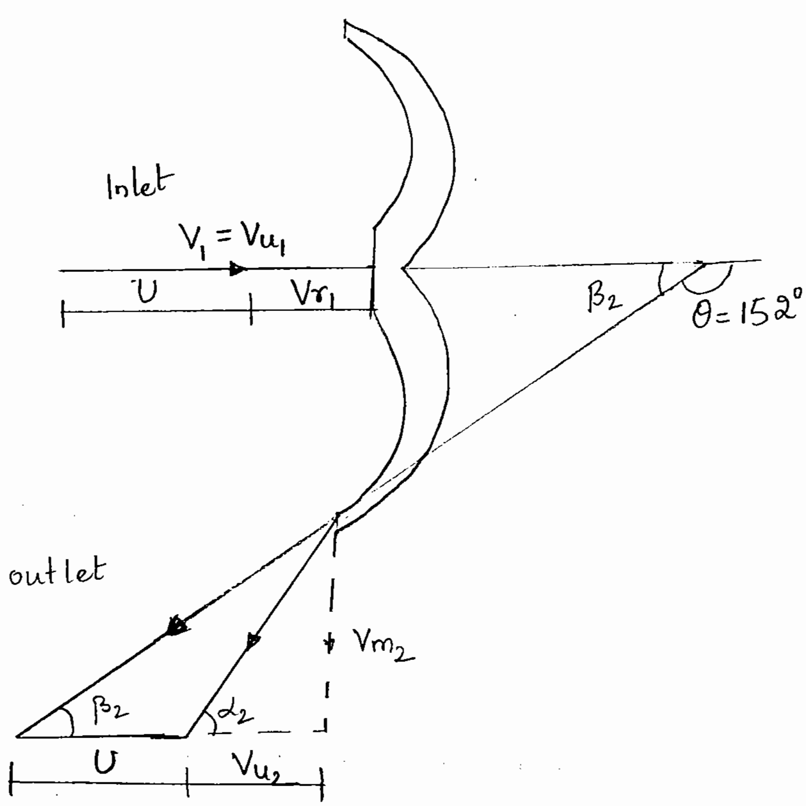
$\boxed{d = 0.1066\,m}$

Number of buckets $Z = \frac{D}{2d} + 15$

$= \frac{2.33}{2(0.1066)} + 15$

$= 25.93$

$\boxed{Z \simeq 26}$



we have $V_1 = V_{u_1} = 179.74 \text{ m/s}$

$$V_{r_1} = V_1 - U = 179.74 - 85.24$$

$$V_{r_1} = 94.5 \text{ m/s}$$

also

$$C_b = \frac{V_{r_2}}{V_{r_1}} = 0.86$$

$$V_{r_2} = 0.86 \times 94.5$$

$$V_{r_2} = 81.27 \text{ m/s}$$

From outlet velocity triangle

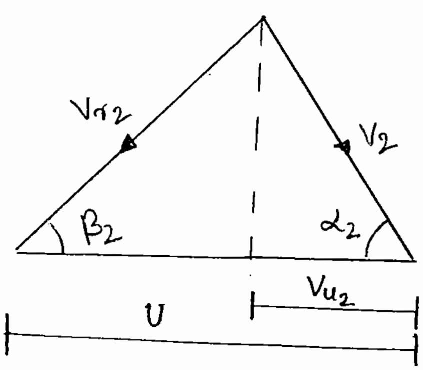
$$\cos\beta_2 = \frac{U + V_{u_2}}{V_{r_2}}$$

$$\cos 28 = \frac{85.24 + V_{u_2}}{81.27}$$

$$V_{u_2} = -13.483 \text{ m/s}$$

"$-$ve sign indicates that the direction of V_{u_2} should be opposite"

Hence outlet velocity triangle becomes



$\therefore V_{u_2} = 13.483 \text{ m/s}$

Tangential force $F_T = \dot{m}\,[V_{u_1} - V_{u_2}]$ $\quad \because V_{u_1} \rightarrow$, $V_{u_2} \rightarrow$

$$\boxed{F_T = 831.285 \text{ N}}$$

3. A double jet Pelton wheel is require to generate 7500 kW when the available head at the base of the nozzle is 400m. The jet is deflected through 165° and the relative velocity of the jet is reduced by 15% in passing over the buckets. Determine a) the diameter of each jet b) the total flow c) the force exerted by the jets in tangential direction. Assume generator efficiency of 95%, overall efficiency of 80%, blade speed ratio of 0.47 and nozzle co-efficient of 0.98.

Given:

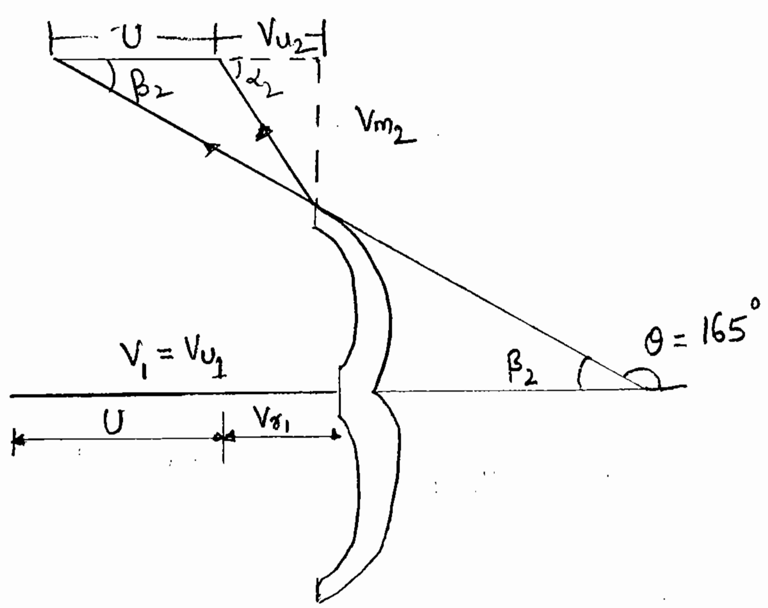
no. of jets $n = 2$, $P_g = 7500\,kW$

$H = 400\,m$, $\beta_2' = 165° = \theta$

$C_b = 85\%$ or 0.85, $\eta_g = 0.95$

$\eta_o = 0.8$, $\phi = 0.47$, $C_v = 0.98$

To find: d, Q, F_T.



we can write o/L vel. Δle at bottom as well as at the top, Both are same.

Generator output power is 7500 kW.

find input power ;

ie $\eta_g = \dfrac{\text{output}}{\text{Input}} = 0.95$

$$\frac{7500}{0.95} = \text{Input}$$

Input (SP) power = 7894.74 kW

[Here, SP is runner output power because η_{mech} (is not given) = 1].

hence $\dot{m}U(V_{u_1} \pm V_{u_2}) = SP = 7894.74\,kW$

also

$$P = \eta_o \frac{\rho g Q H}{1000}.$$

$$7894.74 = 0.8 \times \frac{1000 \times 9.81 \times Q \times 400}{1000}$$

Total discharge $\boxed{Q = 2.515\ m^3/s}$

also.

$$Q = n \cdot q = n \frac{\pi}{4} d^2 V_1$$

$$U = \phi\sqrt{2gH} = 41.64\ m/s.$$

$$V_1 = C_v\sqrt{2gH} = 86.82\ m/s.$$

$$\Rightarrow \quad 2.515 = 2\frac{\pi}{4} \cdot d^2 \times 86.82$$

jet diameter,

From velocity triangle

$$V_1 = V_{u_1} = 86.82\,m/s$$

also $$\cos\beta_2 = \frac{U + V_{u_2}}{V_{r_2}}$$

$$V_{r_1} = V_1 - U = 86.82 - 41.64 = 45.18\,m/s.$$

but $$C_b = \frac{V_{r_2}}{V_{r_1}}.$$

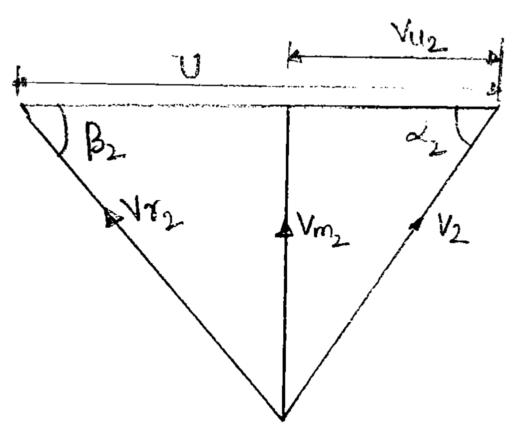
$$V_{r_2} = 45.18 \times 0.85 = 38.403\,m/s$$

$$\cos(180-165) = \frac{41.64 + V_{u_2}}{38.403}$$

$$V_{u_2} = -4.546\,m/s$$

This Indicates, direction should be change

ie



new o/c vel Δle.

Tangential force $F_T = \dot{m}[V_{u_1} - V_{u_2}] = \rho Q[V_{u_1} - V_{u_2}]$

$V_{u_1} \rightarrow$

$V_{u_2} \rightarrow$

$$= 1000 \times 2.515[86.82 - 4.546]$$

$$\boxed{F_T = 206.92\,kN}$$

4. Following data refers to a pelton wheel, gross head 500m, water supply (penstock) diameter 1m, length of the penstock 3.5km. co-efficient of friction. $f = 0.006$, jet diameter 18cm. jet deflection angle 165°, 15% friction on the bucket, peripheral velocity of bucket is 0.46 times the absolute velocity of jet leaving the nozzle, mechanical efficiency 85%. Calculate a) the powered by the runner b) the power at the shaft c) the hydraulic efficiency and d) the overall efficiency.

Given

$H_g = 500m$

dia of penstock $d_p = 1m$

length of penstock $L_p = 3500m$

friction co-efficient $f = 0.006$

jet diameter $d = 18cm = 0.18m$

$\theta = 165°$, $\beta_2 = 15°$, $C_b = 0.85$

$\phi = 0.46$. $\eta_{mech} = 0.85$

To find P, SP, η_H, η_o

we have continuity equation.

$$Q_p = Q_{nozz}.$$

$$\frac{\pi}{4} d_p^2 V_p = \frac{\pi}{4} d^2 V_1$$

$$\Rightarrow \quad V_p = V_1 d^2 \quad \text{— (1)}$$

we have Gross head = Net head + head loss

$$H_g = H + h_f$$

ie $$500 = \frac{V_1^2}{2g} + \frac{4fLV_p^2}{2gd_p}$$

$$500 = \frac{V_1^2}{2g} + \frac{4fLV_1^2 d^4}{2g \times 1} \qquad \because V_p = V_1 d$$

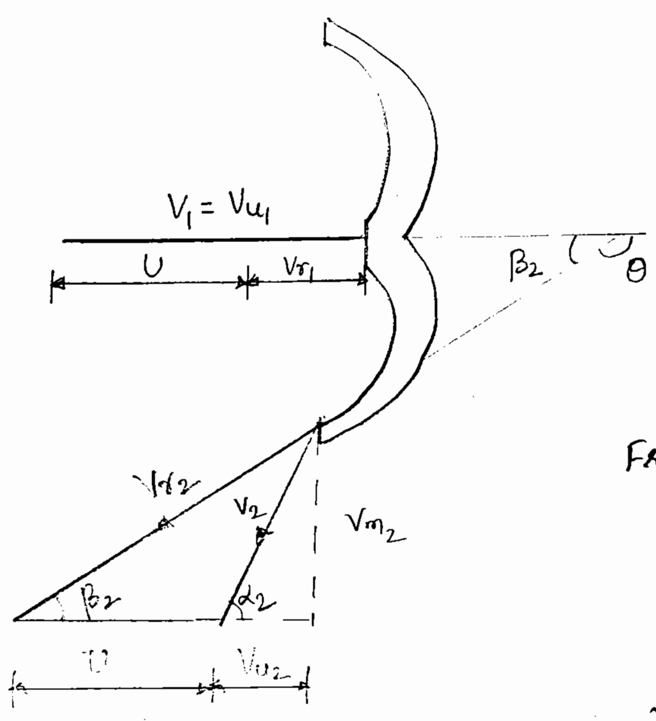
$$500 = \frac{V_1^2}{2 \times 9.81} + \frac{4(0.006)(3500)V_1^2(0.18)^4}{2 \times 9.81}$$

$$\underline{V_1 = 94.95 \, m/s}$$

also $\phi = \frac{U}{V_1} = 0.46$

$U = 0.46 \times 94.95$

$\underline{U = 43.68 \, m/s}$



From inlet velocity triangle

$V_1 = V_{u_1} = 94.95 \, m/s$

$V_{r_1} = V_1 - U = 94.95 - 43.68$

$V_{r_1} = \underline{51.27} \, m/s$

From outlet velocity triangle,

$$C_b = \frac{V_{r_2}}{V_{r_1}} = 0.85 = \frac{V_{r2}}{51.27}$$

$$\underline{V_{r_2} = 43.58\ m/s}$$

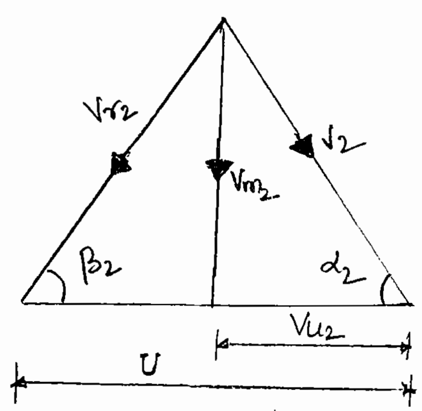
$$\cos\beta_2 = \frac{u + V_{u2}}{V_{r2}}.$$

$$u + V_{u_2} = 42.095$$

$$V_{u_2} = 42095 - 43.68$$

$$\boxed{V_{u_2} = -1.585\ m/s}$$

Hence outlet velocity triangle becomes



Total discharge $Q = \frac{\pi}{4} d_p^2 V_p = \frac{\pi}{4} d_p^2 \cdot V_1 d^2$

$$= \frac{\pi}{4} \times 1^2 \times 94.95 \times 0.18^2$$

$$\boxed{Q = 2.416\ m^3/s}$$

Power developed by runner is

$$P = \dot{m}(V_{u_1} \pm V_{u_2})$$

$P = \rho Q U (V_{u_1} - V_{u_2})$ $V_{u_1} \rightarrow$ $V_{u_2} \rightarrow$

$= 1000 \times 2.416 (94.95 - 1.585) \times 43.68$

$$\boxed{P = 9853 \text{ kW}}$$

also Mechanical efficiency $\eta_{mech} = \dfrac{\text{Shaft power}}{\text{Power by runner}}$

$$\Rightarrow \quad 0.85 = \frac{SP}{9853}$$

$$\boxed{SP = 8375 \text{ kW}}$$

we know, Hydraulic efficiency

$$\eta_H = \frac{\text{power by runner}}{\text{Power available at inlet}} = \frac{P}{\rho g Q H}$$

$$= \frac{9853}{\rho Q V_1^2/2}$$ where $H = V_1^2/2g$

$$= \frac{9853}{1000 \times 2.416 \times \frac{94.95^2}{2}}$$

$$\boxed{\eta_H = 90.47\%}$$

Overall efficiency $\eta_o = \eta_H \cdot \eta_{mech} \cdot \eta_v = 0.9047 \times 0.85 \times 1$

$$\boxed{\eta_o = 76.9\%}$$

5. Design a Pelton wheel to run under a head of 60m at 200rpm while the discharge available is 200lit/s. Assume overall efficiency to be 85%, coefficient of velocity to be 0.98 and speed ratio 0.46.

Given $H = 60m$, $N = 200rpm$

$Q = 200\,lit/s = 0.2m^3/s$

$\eta_0 = 0.85$, $C_v = 0.98$, $\phi = 0.46$

absolute velocity $V_1 = C_v\sqrt{2gH}$

$= 0.98\sqrt{2 \times 9.81 \times 60}$

$\underline{V_1 = 33.62\,m/s}$

Tangential speed $U = \phi\sqrt{2gH}$

$\underline{U = 15.78\,m/s}$

Now, $U = \dfrac{\pi D N}{60}$

$15.78 = \dfrac{\pi \times D \times 200}{60}$

$\boxed{D = 1.507m}$

power developed $P = \eta_0 \dfrac{\rho g Q H}{1000}$

$P = 0.85 \dfrac{9.81 \times 10^3 \times 0.2 \times 60}{1000}$

$\boxed{P = 100.062kw}$

Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$

$N_s = 11.98 \leq 1 \Rightarrow$ Single jet $\underline{\underline{n=1}}$.

We know $Q = nq$

$$0.2 = 1 \times \frac{\pi}{4} d^2 V_1$$

$$0.2 = \frac{\pi}{4} d^2 \times 33.62$$

jet dia $\boxed{d = 0.087 m}$

Width of the bucket $W = 3d = 0.261$

$$\boxed{W = 0.261 m}$$

Depth of the bucket $t = 0.6d = 0.6 \times 0.087$

$$\boxed{t = 0.0522 m}$$

Length of the bucket $L = 2.5d = 2.5 \times 0.087$

$$\boxed{L = 0.2175 m}$$

Number of buckets

$$Z = \frac{D}{2d} + 15$$

$$= \frac{1.507}{2 \times 0.087} + 15$$

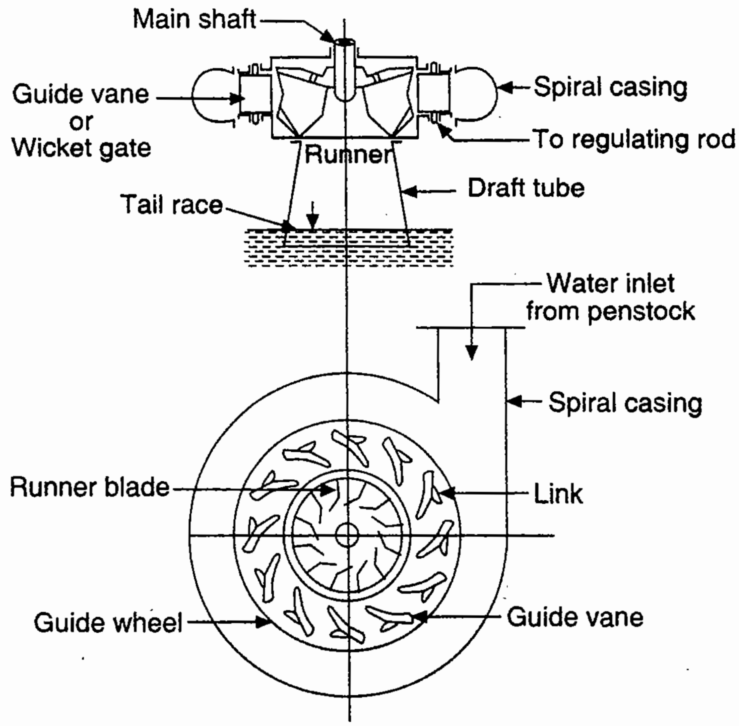
$$= 23.66$$

$$\boxed{Z \simeq 24}$$

Jet ratio $m = \frac{D}{d} = \frac{1.507}{0.087}$

$$m = \underline{\underline{17.32}}$$

Francis Turbine



Francis turbine and its main components.

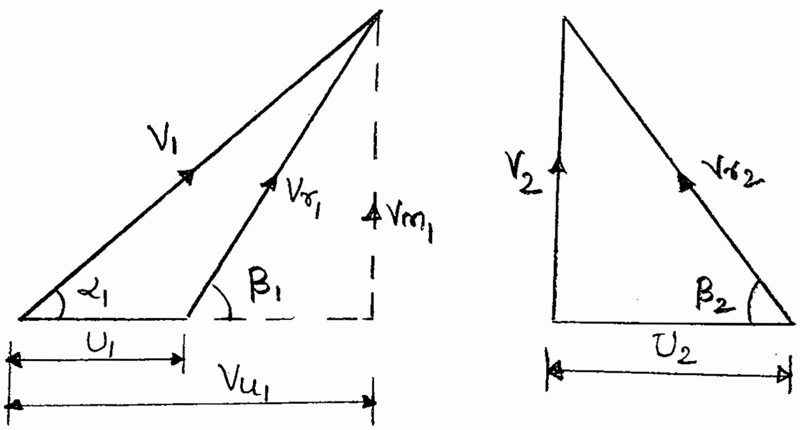
It is a reaction turbine working under medium head and handling medium quantity of water. The specific speed is in the range of 51 rpm to 255 rpm.

The water flows from the reservoir to the turbine through the penstock and feeds water to a row of fixed blades through casing. These fixed blades convert a part of the pressure energy into kinetic energy before the water enters the runner. Thus, water possessing pressure and kinetic energy enters the runner vanes in the radial direction and leaves in the axial direction. Thus, it is a mixed flow. The static pressure of the water at the inlet to the runner is higher than that at the exit. The pressure energy of water is gradually changed into kinetic energy as water flows over the vanes.

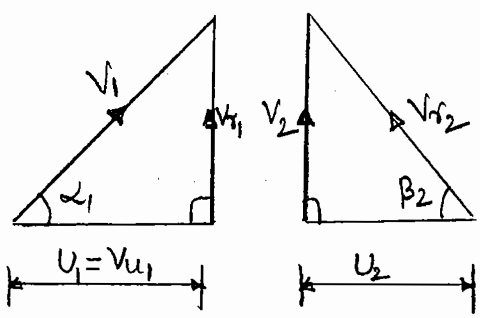
Velocity triangles

"Inward flow reaction turbine with radial discharge".

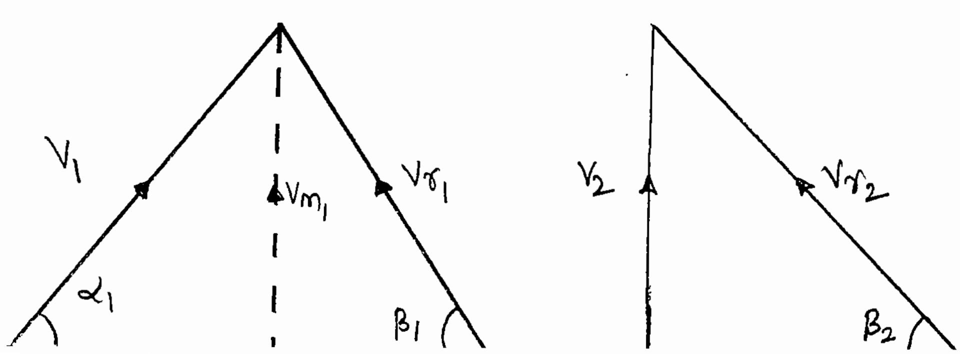
1. Slow speed. inlet blade angle $\beta_1 < 90^\circ$. [ie $V_{u_1} > U_1$]



2. Minimum Speed, $\beta_1 = 90^\circ$ [ie $V_{u_1} = U_1$]



3. High Speed. $\beta_1 > 90^\circ$ (ie $V_{u_1} < U_1$)



Design Parameters

1. Speed ratio $\phi = \frac{U_1}{\sqrt{2gH}}$, $U_1 = \frac{\pi D N}{60} = \dots m/s$

2. Flow ratio , $\psi = \frac{V_{m_1}}{\sqrt{2gH}}$

3. Quantity of water flowing

$$Q = \pi D_1 B_1 V_{m_1} = \pi D_2 B_2 V_{m_2}$$

where D_1, D_2 Diameters of runner at inlet and outlet.

B_1, B_2 width of runner at inlet and outlet.

4. Considering number of vanes and thickness of each vane.

$$Q = (\pi D_1 - n t_1) B_1 V_{m_1} = (\pi D_2 - n t_2) B_2 V_{m_2}$$

where n = number of vanes

t = thickness of ~~ea~~ each vane.

5. Considering the discharge at the draft tube.

$$Q = \frac{\pi}{4} d_3^2 V_3$$

where d_3 = diameter of draft tube at inlet of draft.

V_3 = Velocity of water at draft tube inlet.

2 An Inward flow reaction turbine has a runner 0.5 m diameter and 7.5 cm wide. The inner diameter is 0.35 m. The effective area of flow is 93% of the gross area and the flow velocity is constant. The guide vane angle is 23°, inlet vane angle is 97° and the outlet vane angle is 30°. Calculate the speed, so that the water enters without shock and the power from supply head of 60m. Assume hydraulic friction losses 10% and mechanical efficiency is 94%. What is the specific speed of the machine ?

Given.

$D_1 = 0.5m \qquad B_1 = 7.5cm = 0.075m$

$D_2 = 0.35m \qquad B_2 =$

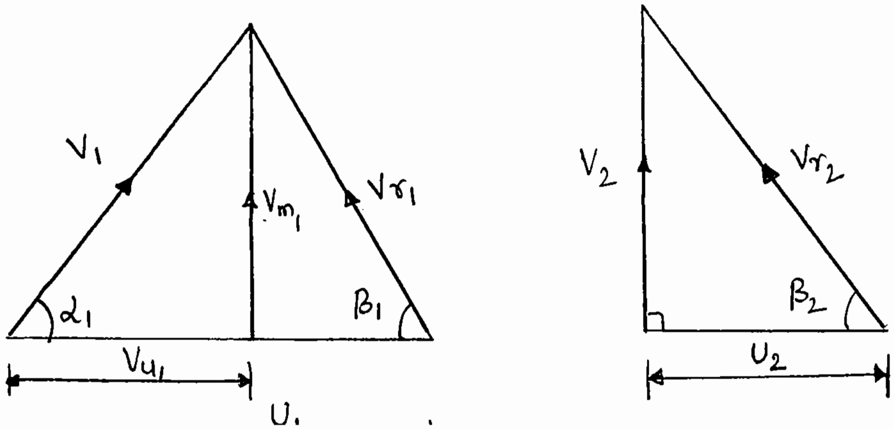
$G = 93\% = 0.93$

$\alpha_1 = 23°, \ \beta_1 = 97°, \ \beta_2 = 30°$

$N = ? \qquad H = 60m$;

$\eta_{mech} = 0.94 \ , \ N_s = ?$

$h_f = 10\% \ H \ , \Rightarrow \ \eta_H = 90\% = 0.9$



From inlet velocity triangle.

$$\tan\alpha_1 = \frac{V_{m_1}}{V_{u_1}}$$

$$V_{u_1} = V_{m_1}/\tan\alpha_1$$

$\boxed{OR}$ $\quad V_{m_1} = V_{u_1}\tan\alpha_1 \quad \text{—— (1)}$

also $\quad \tan\beta_2 = \dfrac{V_2}{U_2}$

$$V_2 = U_2 \tan\beta_2 \quad \text{—— (2)}$$

but $\quad V_{m_1} = V_{m_2} = V_2$

from (1) and (2)

$$V_{u_1}\tan\alpha_1 = U_2\tan\beta_2 \quad \text{—— (3)}$$

we know $\quad U_1 = \dfrac{\pi D_1 N}{60} \quad \text{—— (a)}$

$$U_2 = \frac{\pi D_2 N}{60} \quad \text{—— (b)}$$

$$\frac{(a)}{(b)} \Rightarrow \frac{D_1}{D_2} = \frac{U_1}{U_2}.$$

$$U_2 = U_1(D_2/D_1) \quad \text{—— (c)}$$

put (c) in (3)

$$V_{u_1}\tan\alpha_1 = U_1\frac{D_2}{D_1}\tan\beta_2.$$

$$V_{u_1}\tan 23 = U_1\frac{0.35}{0.5}\tan 30$$

$$\boxed{V_{u_1} = 0.952\,U_1} \quad \text{—— (4)}$$

we know $\quad \eta_H = \dfrac{He}{H} = \dfrac{60-6}{60} = 0.9 = \dfrac{U_1 V_{u_1}}{gH}$

$$\frac{U_1 V_{u1}}{g H_g} = 0.9$$

$$U_1 V_{u_1} = 0.9 \times 9.81 \times 60 = 529.74 \text{ J/kg}$$

also $V_{u_1} = 0.952 U_1$

$$\Rightarrow \boxed{\begin{array}{c} V_{u_1} = 22.43 \text{ m/s} \\ \hline U_1 = 23.62 \text{ m/s} \end{array}}$$

$$V_{m_1} = V_{m_2} = V_2 = V_{u_1} \tan\alpha_1 = 22.43 \times \tan 23$$

$$V_{m_1} = V_2 = V_{m_2} = \underline{\underline{9.52 \text{ m/s}}}$$

Discharge, $Q = C \pi D_1 B_1 V_{m_1}$

$$Q = 0.93 \pi \times 0.5 \times 0.075 \times 9.52$$

$$\underline{\underline{Q = 1.043 \text{ m}^3/\text{s}}}$$

Speed of turbine, $N = \frac{U_1 \times 60}{\pi D_1} = \frac{23.62 \times 60}{\pi \times 0.5}$ $\quad \therefore U_1 = \frac{\pi D_1 N}{60}$

$$\boxed{N = 901.5 \text{ rpm}}$$

power output $P = \eta_o \frac{\rho g Q H}{1000} = \eta_H \cdot \eta_{mech} \frac{\rho g Q H}{1000}$

$$= 0.9 \times 0.94 \times \frac{1000 \times 9.81 \times 1.043 \times 60}{1000}$$

$$\boxed{P = 519.4 \text{ kW}}$$

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{101.5\sqrt{5174}}{60^{5/4}}$

$$\underline{\underline{N_s = 123}}$$

3. A dam is proposed to be built for which a Francis turbine is required to be designed. The design head is 16m and the design flow rate is 8 m³/s. The speed is to be 250 rpm. An overall efficiency of 0.9, hydraulic efficiency of 0.95, a speed ratio of 0.76 and flow ratio of 0.35 may be assumed. Obtain all the salient dimensions, blade angles and guide vane angles. Inner diameter is half the outer diameter and discharge does not have any whirl component. Neglect vane thickness.

Given,

Francis turbine

$H = 16m$, $Q = 8 m^3/s$

$N = 250\ rpm$

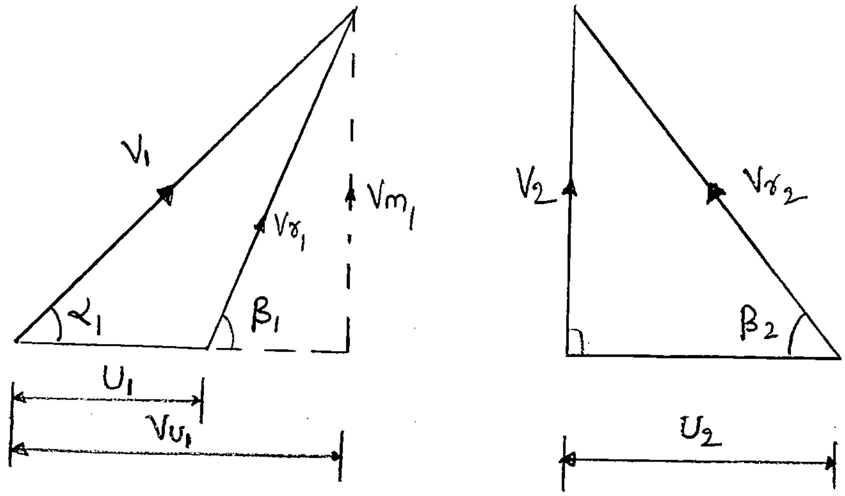
$\eta_o = 0.9$, $\eta_H = 0.95$

$\phi = 0.76$, Speed ratio

Flow ratio $\psi = 0.35$

$D_2 = 0.5 D_1$ $\quad\because$ (Francis) Inward flow $\to$ o/L diameter D_1, Inner dia. D_2

$V_{u_2} = 0$,



we know,

$$U_1 = \phi\sqrt{2gH}$$

$$= 0.76\sqrt{2 \times 9.81 \times 16}$$

$$\boxed{U_1 = 13.465 \text{ m/s}}$$

also.

$$Vm_1 = \psi\sqrt{2gH}$$

$$= 0.35\sqrt{2 \times 9.81 \times 16}$$

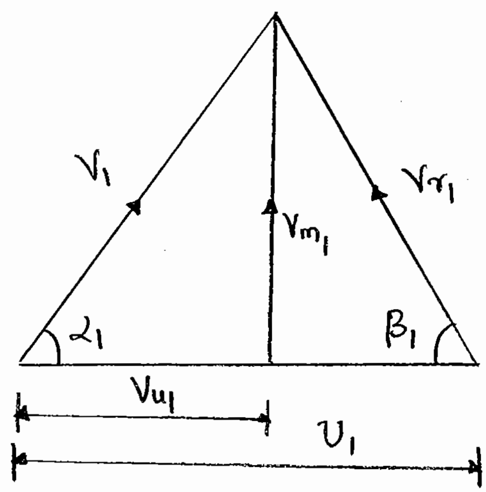
$$\boxed{Vm_1 = 6.2 \text{ m/s}}$$

we have, hydraulic efficiency $\eta_H = \frac{Vu_1 U_1}{gH}$, $\because Vu_2 = 0$

$$0.95 = \frac{Vu_1 \cdot 13.465}{9.81 \times 16}$$

$$\boxed{Vu_1 = 11.074 \text{ m/s}}$$

Since V_{u_1} is less than U_1, hence inlet velocity triangle becomes



From velocity triangle

$$V_1^2 = V_{m_1}^2 + V_{u_1}^2 = 6.2^2 + 11.074^2$$

$$\boxed{V_1 = 12.69 \text{ m/s}}$$

again $$V_{r_1}^2 = V_{m_1}^2 + (U_1 - V_{u_1})^2$$

$$= 6.2^2 + (13.465 - 11.074)^2$$

$$\boxed{V_{r_1} = 6.645 \text{ m/s}}$$

also $$\tan\alpha_1 = \frac{V_{m_1}}{V_{u_1}} = \frac{6.2}{11.074}$$

$$\alpha_1 = \tan^{-1}(0.5599)$$

$$\boxed{\alpha_1 = 29.24^\circ}$$

$$\tan\beta_1 = \frac{V_{m_1}}{U_1 - V_{u_1}} = \frac{6.2}{13.465 - 11.074}$$

$$\boxed{\beta_1 = 68.91^\circ}$$

By assuming vane width is equal ie $B_1 = B_2$

we know

$$Q = \pi D_1 B_1 V_{m_1} \quad — (1)$$

but $U_1 = \frac{\pi D_1 N}{60}$

$$13.465 = \frac{\pi D_1}{60} \times 250$$

$$\boxed{D_1 = 1.029\ m}$$

$$\Rightarrow \boxed{D_2 = 0.5143\ m}$$

(1) $\Rightarrow$

$$8 = \pi (1.029) \times B \times 6.2 \qquad \because B_1 = B_2$$

$$\boxed{B = 0.3991\ m}$$

also $Q = \pi D_2 B_2 V_{m_2}$

$$8 = \pi (0.5143)(0.3991) V_{m_2}$$

$$\boxed{V_{m_2} = V_2 = 12.41\ m/s}$$

From outlet velocity triangle.

$$\tan\beta_2 = \frac{V_2}{U_2} = \frac{12.41}{\left(\frac{\pi D_2 N}{60}\right)}$$

$$\boxed{\beta_2 = 61.52^\circ}$$

power $P = \eta_o \frac{\rho g Q H}{1000}$

$$= \frac{0.9 \cdot 9810 \times 8 \times 16}{1000}$$

$$P = 1130.112\ kW$$

4. The internal and external diameters of an inward flow reaction turbine are 1.2m and 0.6m respectively. The head on turbine is 22m and velocity of flow through the runner is constant and is equal to 2.5m/s. The guide blade angle is 10° and the runner vanes are radial at inlet. If the discharge at outlet is radial. Find i) Speed of turbine ii) Vane angle at outlet iii) Hydraulic efficiency iv) Draw velocity triangle.

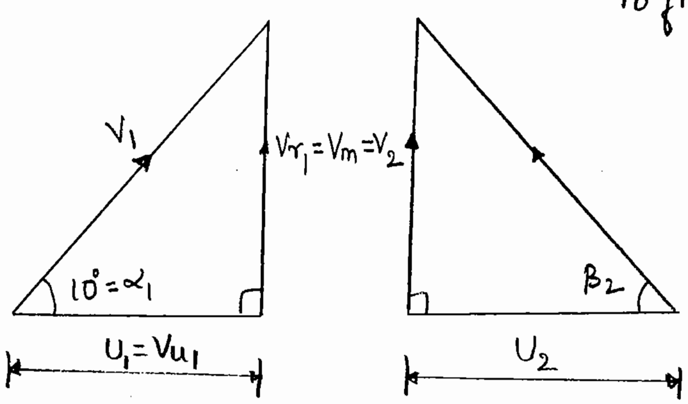
Given

$D_1 = 1.2m$, $D_2 = 0.6m$

$H = 22m$, $V_{m_1} = V_{m_2} = V_2 = 2.5 m/s$

$\alpha_1 = 10^\circ$ $\beta_1 = 90^\circ$, $\alpha_2 = 90^\circ$

To find: $N, \beta_2 \cdot \eta_H$



Since V_{r_1} is $\perp^{er}$, $V_{r_1} = V_m = V_2$.

From inlet velocity triangle

$$\tan\alpha_1 = \frac{V_2}{U_1}$$

$$\tan 10 = \frac{2.5}{U_1}$$

$$\boxed{U_1 = 14.18 m/s}$$

also $$U_1 = \frac{\pi D_1 N}{60}$$

$$14.18 = \frac{\pi \times 1.2 \times N}{60}$$

$$\boxed{N = 225.68 \text{ rpm}}$$

again $$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 225.68}{60}$$

$$\boxed{U_2 = 7.09 \text{ m/s}}$$

From outlet velocity triangle.

$$\tan\beta_2 = \frac{V_2}{U_2} = \frac{2.5}{7.09}$$

$$\boxed{\beta_2 = 19.42^\circ}$$

we know hydraulic efficiency

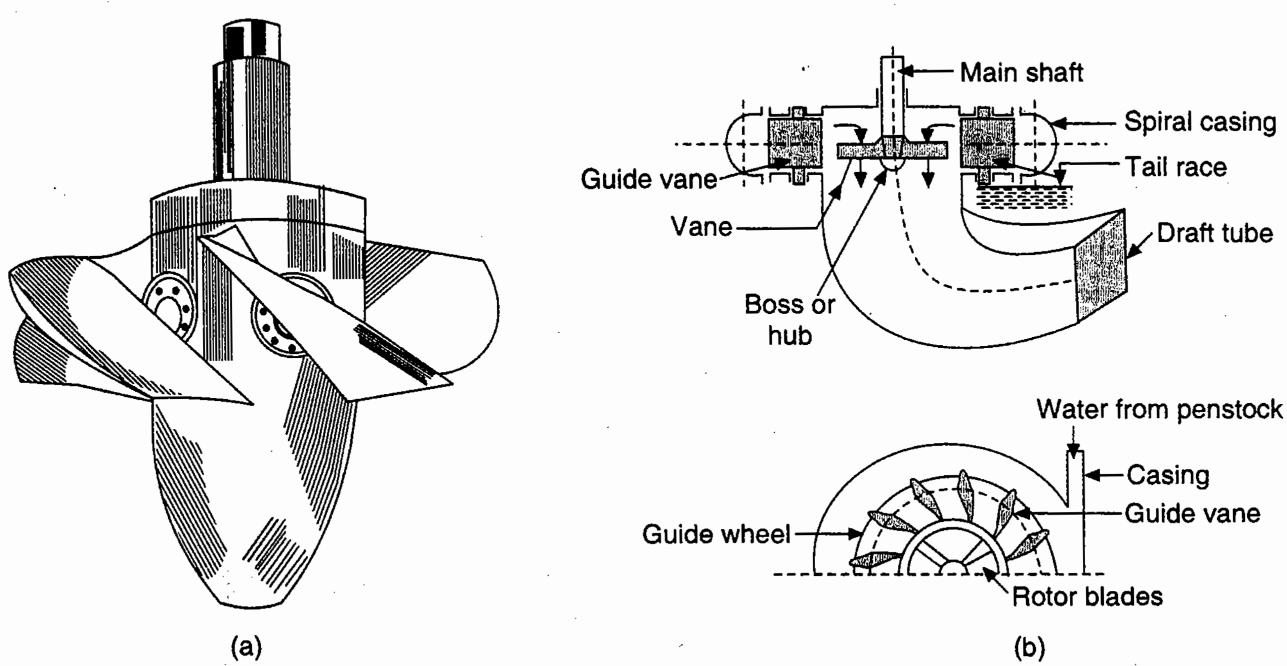
$$\eta_H = \frac{U_1 V_{u1}}{gH} = \frac{U_1^2}{gH} = \frac{14.18^2}{9.81 \times 22}$$

$$\boxed{\eta_H = 93.17\%}$$

Kaplan turbine

The Kaplan turbine is an axial flow reaction turbine in which the flow is parallel to the axis of the shaft as shown. This is mainly used for large quantity of water and for very low heads for which the specific speed is high.

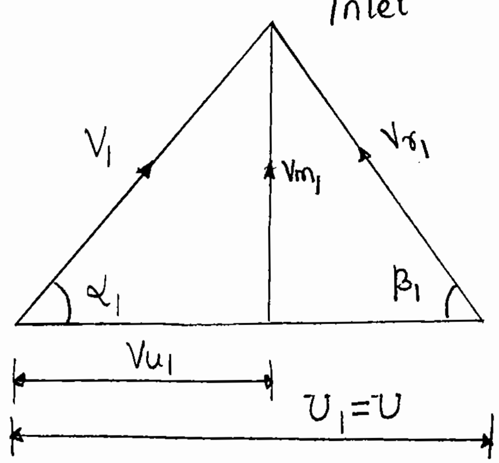
The runner of the Kaplan turbine looks like a propeller of a ship. Therefore sometimes it is also called as propeller turbine. At the exit of the Kaplan turbine the draft tube is connected to discharge water to the tail race.



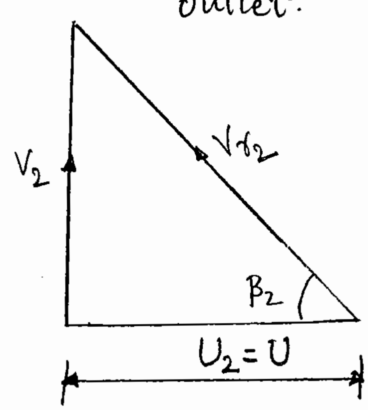
Kaplan turbine: (a) Runner. (b) Turbine with components.

Velocity triangles

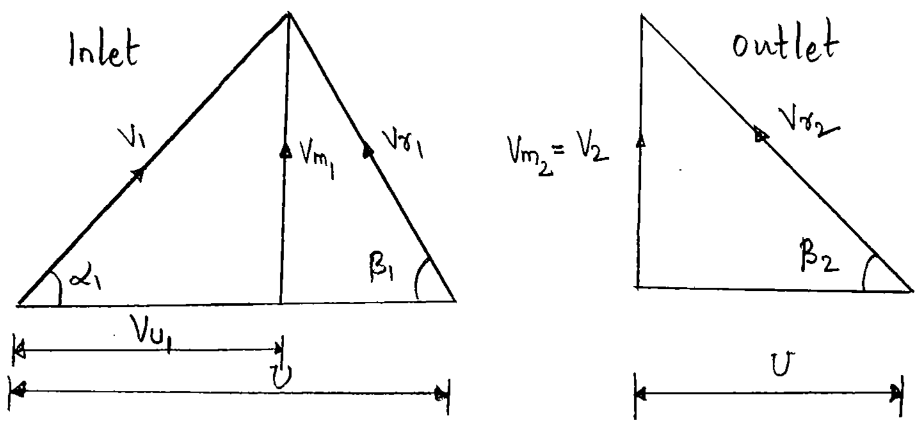
Inlet



outlet.



1. A Kaplan turbine working under a head of 15m develops 7350kW. The outer diameter of the runner is 4m and hub diameter is 2m. The guide blade angle at the extreme edge of the runner is 30°. The hydraulic and the overall efficiency of the turbine are 90% and 85% respectively. If the velocity of the whirl is zero at outlet, determine 1) Runner vane angle at inlet and outlet at the extreme edge of the runner (2) Speed of the turbine.



Given: $H = 15m$, $P = 7350 kW$

$D = 4m$ $\quad d = 2m$

$\alpha_1 = 30°$, $\eta_H = 0.9$, $\eta_o = 0.85$

$V_{u_2} = 0$

To find $\beta_1 \cdot \beta_2 \cdot N$

we have
$$P = \eta_o \frac{\rho g Q H}{1000}$$

$$7350 = 0.85 \frac{9810 \times Q \times 15}{1000}$$

$$Q = \underline{58.7636 \ m^3/s}$$

also $Q = \frac{\pi}{4}(D^2 - d^2)\, V_m$

$$V_{m_1} = V_{m_2} = V_2 = \frac{Q}{\frac{\pi}{4}(D^2 - d^2)}$$

$$= \frac{58.7636}{\frac{\pi}{4}[4^2 - 2^2]}$$

$$\boxed{V_m = V_2 = 6.235\ m/s}$$

From inlet velocity triangle,

$$\tan\alpha_1 = \frac{V_{m_1}}{V_{u_1}}$$

$$V_{u_1} = \frac{6.235}{\tan 30}$$

$$\underline{V_{u_1} = 10.8\ m/s}$$

also Hydraulic efficiency $\eta_H = \frac{U_1 V_{u_1}}{gH}$

$$0.9 = \frac{U_1 \cdot 10.8}{9.81 \times 15}$$

$$\underline{\underline{U = 12.2625\ m/s}}$$

we know $U = \frac{\pi D N}{60}$

$$N = \frac{12.2625 \times 60}{\pi \times 4}$$

$$\boxed{N = 58.55\ rpm}$$

$\tan\beta_1 = \dfrac{}{U - V_{u_1}}$

$$= \frac{6.235}{12.2625 - 10.8}$$

$$\boxed{\beta_1 = 76.8^\circ}$$

$$\tan\beta_2 = \frac{V_2}{U} = \frac{6.235}{12.2625}$$

$$\boxed{\beta_2 = 26.95^\circ}$$

Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{58.55\sqrt{7350}}{15^{5/4}}$

$$\boxed{N_s = 170.04}$$

2. A Kaplan turbine develops 9000kW under a head of 10m. Overall efficiency of the turbine is 85%. The speed ratio based on outer diameter is 2.2 and flow ratio 0.66. Diameter of the boss is 0.4 times the outer diameter of the runner. Determine the diameter of the runner, boss diameter and specific speed of the runner.

Given.

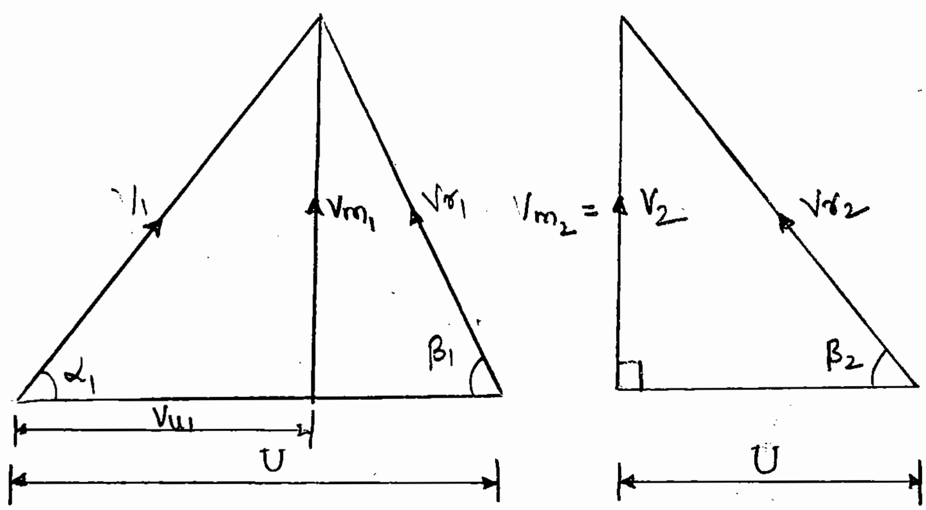
$P = 9000 kW$

$H = 10m$, $\eta_o = 0.85$

$\phi = 2.2$, $\psi = 0.66$

$\frac{d}{D} = 0.4$

To find d, D, N_s



we have

$$\phi = \frac{U}{\sqrt{2gH}}$$

$$U = 2.2\sqrt{2 \times 9.81 \times 10}$$

$$U = \underline{\underline{30.82}}\ m/s$$

$$\psi = \frac{V_m}{\sqrt{2gH}}$$

$$V_m = 0.66\sqrt{2 \times 9.81 \times 10}$$

$$\underline{\underline{V_m = 9.245\, m/s}}$$

power $p = \eta_0 \dfrac{\delta g Q H}{1000}$

$$9000 = 0.85 \quad \frac{1000 \times 9.81 \times Q \times 10}{1000}$$

$$Q = 107.933 \ m^3/s.$$

we know

$$Q = \frac{\pi}{4}\left(D^2 - d^2\right) V_m$$

$$107.933 = \frac{\pi}{4}\left[D^2 - (0.4D)^2\right] 9.245 \qquad \because \frac{d}{D} = 0.4$$

$\boxed{D = 4.207 m}$ diameter of runner

$\Rightarrow$ $\boxed{d = 1.6828 m}$

we have $v = \dfrac{\pi D N}{60} \Rightarrow 30.82 = \dfrac{\pi (4.207) N}{60}$

$$\underline{N = 139.91 \ rpm}$$

Specific Speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{139.91\sqrt{9000}}{10^{5/4}}$

$$\boxed{N_s = 746.4}$$

3. Kaplan turbine produces 40000HP under a head of 9.6m, while running at 65.2 rpm and discharge 350 m^3/s of water. The tip diameter is 7.4 m and ratio $\frac{d}{D}$ is 0.432.

Compute (1) Turbine efficiency

(2) Speed ratio and flow ratio

(3) Specific speed.

<u>Given,</u> $H = 9.6m$, $Q = 350\,m^3/s$

$N = 65.2\,rpm$, $D = 7.4\,m$.

$d/D = 0.432$. , $P = 40000HP = 29828\,kW$

$$\Rightarrow \frac{d}{7.4} = 0.432$$

$$\boxed{d = 3.2\,m}$$

To find: η_t, ϕ, ψ, N_s

we know $P = \eta_o \frac{\rho g Q H}{1000}$.

$$29828 = \eta_o \cdot \frac{1000 \times 9.81 \times 350 \times 9.6}{1000}.$$

$\eta_o = \eta_t = 0.905$

$$\boxed{\eta_t = 90.5\%}$$

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{65.2\sqrt{29828}}{9.6^{5/4}}$

$\boxed{N_s = 666.4}$ or $N_s = 666.38$ rpm

We know $U = \frac{\pi D N}{60} = \frac{\pi \times 1.4 \times 65.2}{60}$

$$U = 25.2625 \text{ m/s}$$

also $Q = \frac{\pi}{4}(D^2 - d^2) V_m$

$$350 = \frac{\pi}{4}(7.4^2 - 3.2^2) V_m$$

$$V_m = 10 \text{ m/s}$$

Speed ratio $\phi = \frac{U}{\sqrt{2gH}} = \frac{25.2625}{\sqrt{2 \times 9.81 \times 9.6}}$

$$\boxed{\phi = 1.8407}$$

Flow ratio $\psi = \frac{V_m}{\sqrt{2gH}} = \frac{10}{\sqrt{2 \times 9.81 \times 9.6}}$

$$\boxed{\psi = 0.73}$$

8

Centrifugal Pumps

Q.1. What is a pump? What is the principle on which a pump works? What are the types of pump?

Ans : The hydraulic machine which converts the mechanical energy into hydraulic energy is called a pump. The hydraulic energy is in the form of pressure energy. The centrifugal pump works on the principle of forced vortex flow which means that when a certain mass of a liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place. The rise in the pressure head at any point of the rotating liquid is proportional to the square of the tangential velocity of the liquid at that point. Thus at the outlet of the impeller where the radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure head. Due to high pressure head, the liquid can be lifted to a high level.

Depending upon the principal of operation there are two important types of pumps.

1. *Centrifugal pump*, also called a turbomachine, in which the mechanical energy is converted to hydraulic energy by a rotating element called impeller.

2. *Reciprocating pump*, also called positive displacement machine, in which the mechanical energy is converted to hydraulic energy by the reciprocating element called piston.

Q.2. Explain with a neat sketch, constructional details and principle of operation of a centrifugal pump. (May 2011)

Ans : The main parts of a centrifugal pump as shown in Fig.8.1, are : 1. Impeller, 2. Casing, 3. Suction pipe with a foot valve and

a strainer and 4. Delivery pump.

1. Impeller : The rotating part of a centrifugal pump is called a impeller. It consists of series of backward curved vanes. The impeller is mounted on a shaft which is connected to the shaft of an electric motor.

2. Casing : The casing is an air tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe. The different types of casings are (a) Volute casing, (b) Vortex casing and (c) Casing with guide blades.

3. Suction pipe with a foot valve and a strainer : It is a pipe whose one end is connected to the inlet of the pump and the other end is dipped into the water in the sump. A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe. The foot valve opens in upward direction only. A strainer is also fitted at the lower end of the suction pipe.

4. Delivery pipe : It is a pipe whose one end is connected to the outlet of the pump and the other end delivers water at a required height.

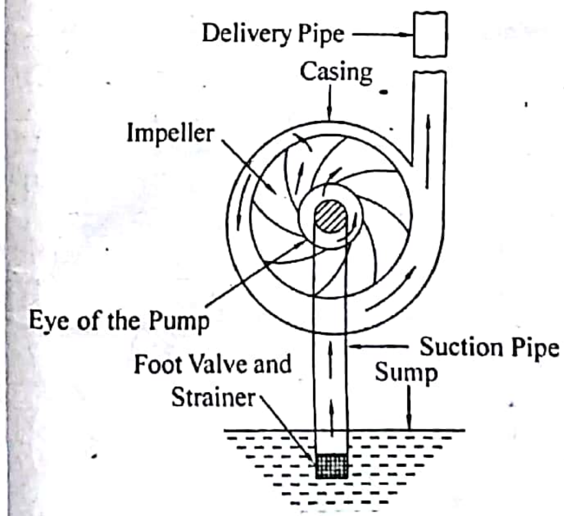


Fig.8.1. Main Parts of a Centrifugal pump.

$$Q = \frac{N_s^2 H_m^{\frac{3}{2}}}{N^2}$$

or
$$N_s = \sqrt{\frac{QN^2}{H_m^{\frac{3}{2}}}} = \frac{N\sqrt{Q}}{H_m^{\frac{3}{4}}}$$

Q.9. What do you mean by multistage pump?

Ans : If a centrifugal pump consists of two or more impellers, then it is called a multistage centrifugal pump. The impellers may be mounted on a single shaft or different shafts. The multistage centrifugal pumps are used for the following two reasons :

1. To obtain a high head in which the impellers are connected in series, and

2. To obtain a large discharge in which the impellers will be connected in parallel.

Q.10. How do you obatin high head from a centrifugal pump?

Ans :

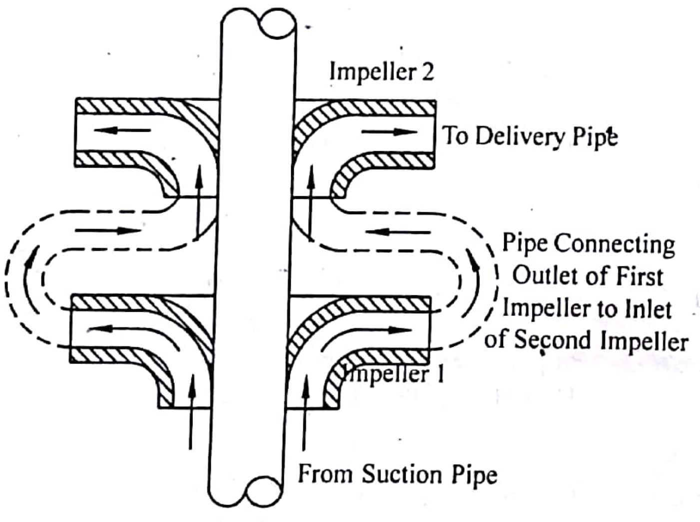


Fig.8.6. Two Stage Pumps with Impellers in series

In order to obtain a high head for water, a number of impellers are mounted in series as shown in Fig.8.6. The water from the suction pipe enters the first impeller at inlet and is discharged at outlet with increased pressure. The water with increased pressure at the outlet of the first impeller is taken to the inlet of the second impeller with the help of a connecting pipe. At the outlet of the second impeller, the pressure of the water will be more than the pressure at the outlet of the first impeller. Thus the pressure of the water will be increased by utilizing more impellers in series.

If H_m is the head developed by each impeller, then the total head developed with 'n' impellers in series is $n \times H_m$, the discharge being the same.

Q.11. How do you obatin high discharge from a centrifugal pump?

Ans : For obtaining high discharge, the pumps should be connected in parallel as shown in Fig.8.7. Each pump lifts the water from the common sump and discharges water to a common pipe to which the delivery pipes of each pump is connected. Each pump is working against the same head.

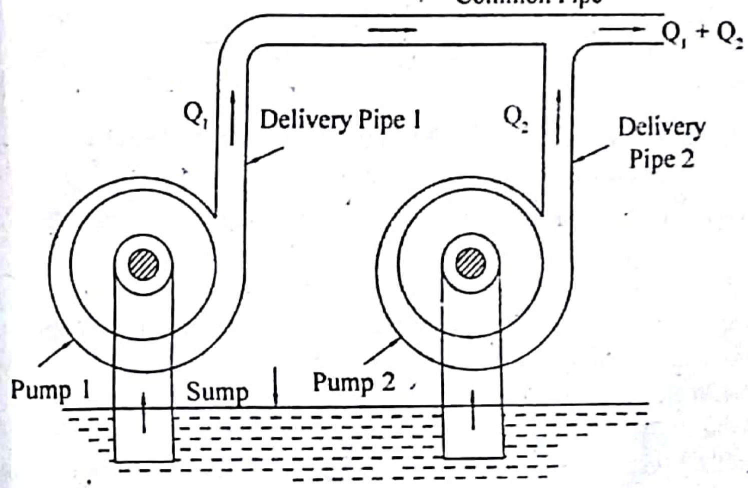


Fig.8.7. Two Stage Pumps with Impellers in Parallel.

Let n = Number of identical pumps arranged in parallel.

Q = Discharge from one pump.

$\therefore$ Total discharge $= n \times Q$

Q.12. What is priming of a centrifugal pump? Why is it necessary?

Ans : The work done by the impeller per unit weight of liquid per unit sec is known as the head generated by the pump. The head generated by the pump is given by the equation, $\frac{1}{g} U_2 V_{w2}$ meter. This equation is independent of the density of the liquid. This means that when pump is running in air, the head generated is in terms of meter of air. If the pump is primed with water, the head generated is in terms of water. But as the density of air is low, the generated head of air in terms of equivalent meter of water head is negligible and hence the water may not be sucked from the pump. To avoid this difficulty, priming is necessary.

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

Q.13. What is cavitation? What are its effect? What are the precautions against cavitation?

Ans : Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure falls below is vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surfaces, above which the liquid is flowing, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced.

The effects are the cavitation are :

1. The metallic surfaces are damaged and cavities are formed on the surfaces.

2. Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.

3. Due to pitting action the surface of the blades become rough and the force exerted by blades on water decreases. Hence the efficiency or work output decreases.

The precautions against cavitation are :

1. The pressure of the flowing liquid in any part of the system should not be allowed to fall below its vapour pressure. For water it is 2.5 m.

2. The special materials or coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials should be used.

Q.14. A centrifugal pump delivers water against a head of 20 m at the rate of 100 lit/s at the speed of 1500 rpm. The impeller diameter is 30 cm and width at outlet is 5 cm. If the manometric efficiency is 80% determine the vane angle at the outlet of the impeller.

Solution : Given :

$H_m = 20$ m, $Q = 100$ lit/s $= 0.1$ m³/s, $N = 1500$ rpm,

$D_2 = 30$ cm $= 0.3$ m, $B_2 = 5$ cm $= 0.05$ m,

$\eta_{man} = 80\% = 0.8.$

The blade velocity at the outlet,

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1500}{60} = 23.56 \text{ m/s}$$

The discharge is given by,

$$Q = \pi D_2 B_2 V_{f2}$$

Therefore velocity of flow at outlet,

$$V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{0.1}{\pi \times 0.2 \times 0.05} = 3.18 \text{ m/s}$$

Net Positive suction head - (NPSH)

The term NPSH is very commonly used in Pump industry

~~Actually the minium~~ Suction

NPSH is defined as the absolute pressure head at the inlet to the Pump minus the vapour pressure head (in absolute units) plus the velocity head

∴ NPSH = Abs pr head at inlet of the pump − vapour pr head (abs unit) + velocity head

$$NPSH = \frac{P_1}{\rho g} - \frac{P_v}{\rho g} + \frac{V_s^2}{2g} \quad \text{—} \ (1)$$

h_s - static head

h_{fs} - frictional head loss in suction pipe.

Absolute pr head at inlet of the pump

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} - \left[\frac{V_s^2}{2g} + h_s + h_{fs}\right] \quad \text{—} \ (2)$$

Substitute (2) in (1)

$$\therefore NPSH = \left[\frac{P_a}{\rho g} - \left(\frac{V_s^2}{2g} + h_s + h_{fs}\right)\right] - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}$$

$$= \frac{P_a}{\rho g} - \frac{P_v}{\rho g} - h_s - h_{fs}$$

$$= H_a - H_v - h_s - h_{fs}$$

where

$$H_a = \frac{P_a}{\rho g} = \text{Atmospheric head}$$

$$H_v = \frac{P_v}{\rho g} = \text{vapour pressure head}$$

$$\therefore NPSH = (H_a - h_s - h_{fs}) - H_v$$

RHS of above eqn is the total ~~head~~ suction head. Hence NPSH is equal to total suction head. Thus NPSH may also be defined as total head required to make the liquid flow through the suction pipe to the pump impeller.

Minimum speed for starting a centrifugal pump

When centrifugal pump is started, It will start delivering liquid only if the pressure rise in the impeller is more than or equal to the manometric head H_m.

For minium starting speed, we must have

$$\frac{u_2^2 - u_1^2}{2g} \geq H_m \quad \text{or} \quad \frac{u_2^2 - u_1^2}{2g} = H_m \quad \text{— ①}$$

WKT $$H_m = \frac{\eta_{mano} V_{u2} u_2}{g} \quad \text{— ②}$$

Substitute ② in ①

$$\frac{u_2^2 - u_1^2}{2g} = \frac{\eta_{mano} V_{u2} u_2}{g}$$

$$\frac{1}{2g}\left[\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2\right] = \eta_{mano} \frac{V_{u2}}{g}\left(\frac{\pi D_2 N}{60}\right)$$

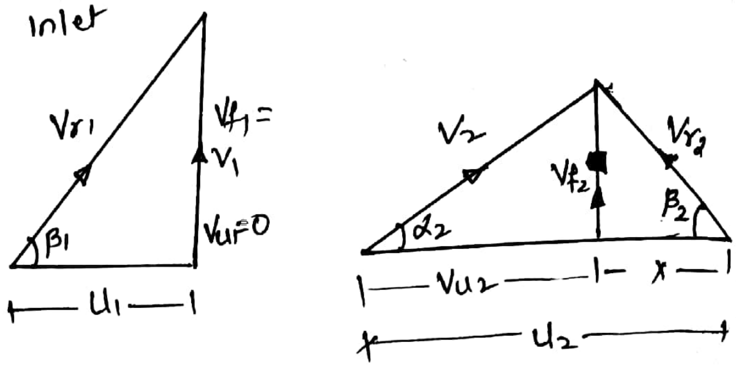
Divide both side by $\frac{\pi N}{60 g}$ we get

$$\frac{\pi N}{120}\left(D_2^2 - D_1^2\right) = \eta_{mano} V_{u2} D_2$$

or $$N = N_{min} = \frac{120 \times \eta_{mano} \times V_{u2} D_2}{\pi \left(D_2^2 - D_1^2\right)}$$

* Show that the pressure rise in the impeller of a CF Pump when the frictional and other losses in the impeller are neglected, is given by $\frac{1}{2g}\left[V_{f_1}^2 + u_2^2 - V_{f_2}^2 \operatorname{cosec}^2\beta_2\right]$ where V_{f_1} and V_{f_2} are the flow velocities at inlet and outlet of the impeller, u_2 = tangential speed of impeller at exit, β_2 = Exit blade angle.

Soln



Applying Bernoulli's eqn between inlet & outlet of a pump neglecting other losses & elevation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \text{WD by Pump} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \frac{u_2 V_{u2}}{g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_2 - P_1}{\rho g} = \frac{u_2 V_{u2}}{g} - \frac{(V_2^2 - V_1^2)}{2g} \quad \text{— (A)} \quad \Rightarrow \frac{P_2 - P_1}{\rho g} = \frac{1}{2g}\left[2u_2 V_{u2} - (V_2^2 - V_1^2)\right] \quad \text{— (A)}$$

From velocity triangle

$V_1 = V_{f_1}$, $V_{u2} = u_2 - V_{f_2}\cot\beta_2$

$$V_2^2 = V_{f_2}^2 + V_{u2}^2 = V_{f_2}^2 + (u_2 - V_{f_2}\cot\beta_2)^2$$

$$V_2^2 = V_{f_2}^2 + u_2^2 + V_{f_2}^2\cot^2\beta_2 - 2u_2 V_{f_2}\cot\beta_2$$

$$\therefore V_2^2 - V_1^2 = V_{f_2}^2 - V_{f_1}^2 + u_2^2 + V_{f_2}^2\cot^2\beta_2 - 2u_2V_{f_2}\cot\beta_2$$

$$= u_2^2 - V_{f_1}^2 + V_{f_2}^2(1+\cot^2\beta_2) - 2u_2V_{f_2}\cot\beta_2$$

$$= u_2^2 - V_{f_1}^2 + V_{f_2}^2\operatorname{cosec}^2\beta_2 - 2u_2V_{f_2}\cot\beta_2 \quad \text{— (B)}$$

Also $u_2 V_{u2} = u_2(u_2 - V_{f_2}\cot\beta_2)$

$$= u_2^2 - u_2V_{f_2}\cot\beta_2 \quad \text{— (C)}$$

Substitute (B) and (C) in (A)

$$\frac{P_2 - P_1}{\rho g} = \frac{1}{2g}\left[2u_2 V_{u2} - (V_2^2 - V_1^2)\right]$$

$$= \frac{1}{2g}\left[2u_2^2 - 2u_2V_{f_2}\cot\beta_2 - u_2^2 + V_{f_1}^2 - V_{f_2}^2\operatorname{cosec}^2\beta_2 + 2u_2V_{f_2}\cot\beta_2\right]$$

$$= \frac{1}{2g}\left[2u_2^2 - \cancel{2u_2V_{f_2}\cot\beta_2} - u_2^2 + V_{f_1}^2 - V_{f_2}\operatorname{cosec}^2\beta_2 + \cancel{2u_2V_{f_2}\cot\beta_2}\right]$$

$$= \frac{1}{2g}\left[V_{f_1}^2 + u_2^2 - V_{f_2}^2\operatorname{cosec}^2\beta_2\right]$$

Scanned by CamScanner

The internal and external diameter of the impeller of a centrifugal pump are 200 mm and 400 mm resp. The pump is running at 1200 rpm. The vane angle of the impeller at inlet and outlet are 20° and 30° resp. The water enters the impeller radially and velocity of flow is constant. Determine the work done by impeller per unit weight of water.

Sol) Internal dia of impeller $D_1 = 0.20\,m$ | $\beta_1 = 20^\circ$

External dia of impeller $D_2 = 0.40\,m$ | $\beta_2 = 30^\circ$

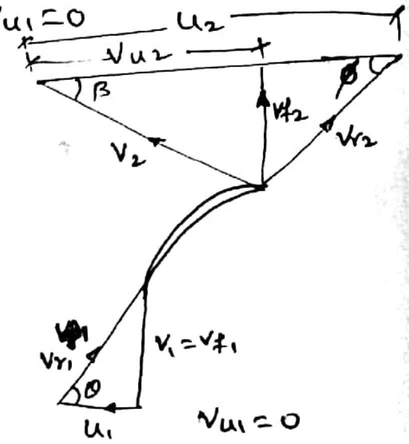
$N = 1200$ rpm

Vane angle at inlet $\theta = 20^\circ = (\beta_1)$

vane angle at outlet $\phi = 30^\circ = \beta_2$

water enters radially i.e, $\alpha_1 = 90^\circ$ & $V_{u1} = 0$

Flow velocity $V_{f1} = V_{f2}$



Tangential velocity of impeller at inlet & outlet

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1200}{60} = 12.56\ m/s$$

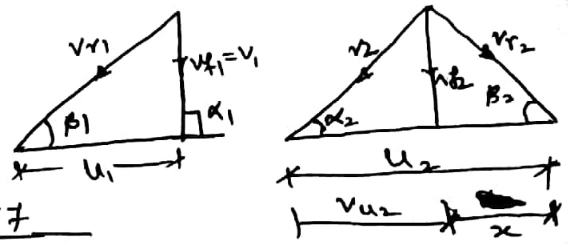
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13\ m/s$$

from fig $\tan\beta_1$ $\tan\theta = \frac{V_{f1}}{u_1} = \frac{V_{f1}}{12.56}$ (inlet velocity triangle)

$\therefore V_{f1} = 12.56 \tan 20^\circ$

$V_{f1} = 4.57\ m/s$

$V_{f2} = V_{f1} = 4.57\ m/s$



from outlet velocity Δle

$\tan\beta_2 = \tan\phi = \frac{V_{f2}}{u_2 - V_{u2}} \Rightarrow \tan 30 = \frac{4.57}{25.13 - V_{u2}}$

$\therefore V_{u2} = 17.915$

$\therefore$ work done by the impeller per kg of water per second is

$$= \frac{1}{g} V_{u2} \times u_2 \Rightarrow WD = \frac{17.215 \times 25.13}{9.81} = 44.1\ Nm\text{-}1N.$$

A Centrifugal pump having outer diameter equal to 2 times the inner diameter and running at 1000 rpm, working under a head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine

a) vane angle at inlet b) WD by impeller on water per sec

c) Manometric efficiency Q5 b

Soln

$N = 1000$ rpm, $H_m = 40$ m, $V_{f1} = V_{f2} = 2.5$ m/s

$\phi (\text{or } \beta_2) = 40°$, $D_2 = 500\,mm = 0.5\,m$, $D_1 = \frac{D_2}{2} = 0.25\,m$

width at outlet $B_2 = 50\,mm = 0.05\,m$

Tangential velocity of impeller at inlet $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09$ m/s

" " " " " outlet $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1000}{60} = 26.18$ m/s

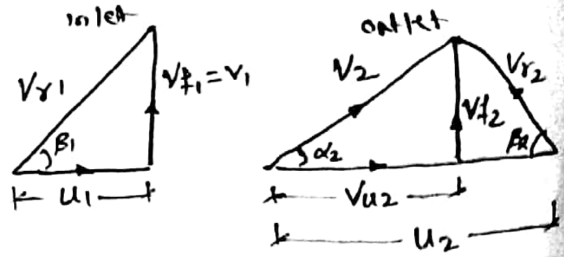
Discharge $Q = A \times V_{f2} \Rightarrow Q = \pi D_2 B_2 \times V_{f2} = \pi \times 0.5 \times 0.05 \times 2.5 = 0.1963\ m^3/s$

a)

Vane angle at inlet θ (or β_1)

From velocity Δ^{le} inlet $\tan\theta = \frac{V_{f1}}{u_1}$

$\tan\theta = \frac{2.5}{13.09}$ $\therefore \theta = 10.81° = \beta_1$ $(\theta = \beta_1)$

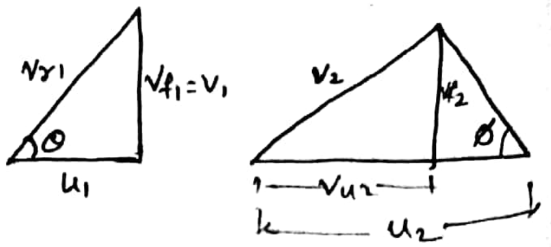


WD by impeller on water per sec

$= \frac{1 \times \omega}{g} \times V_{u2} u_2$

$= \frac{\rho g Q}{g} \times V_{u2} \times u_2$

$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{u2} \times 26.18$



also from outlet velocity triangle $(\phi = \beta_2)$

$\tan\phi = \frac{V_{f2}}{u_2 - V_{u2}} = \frac{2.5}{26.18 - V_{u2}}$

$\therefore \tan 40 = \frac{2.5}{26.18 - V_{u2}}$ $\therefore V_{u2} = 23.2$ m/s

$\therefore$ WD $= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$

b)

$= 119227.9$ Nm/s.

c)

Manometric efficiency $\eta_{man} = \frac{g H_m}{V_{u2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = 64.6\%$

Assignment.
NCQ - Cavitation
priming
pump

The outer dia of an impeller of a centrifugal pump is 400 mm and outlet width 50 mm. The pump is running at 800 rpm and is working against a total head of 15 m, the vanes angles at outlet is 40° and manometric efficiency is 75%. Determine ① Velocity of flow at outlet (V_{f_2})

② Velocity of water leaving the vane (V_2)

③ Angle made by the absolute velocity at outlet with the direction of motion at outlet

④ Discharge

Soln

Outer dia $D_2 = 400\,mm = 0.4\,m$

width at outlet $B_2 = 50\,mm = 0.05\,m$

$N = 800\,rpm$

$H_m = 15\,m$

Vane outlet angle $\beta_2 = 40^\circ$

Manometric efficiency $\eta_{man} = 0.75$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60} = 16.75\,m/s$$

$$\eta_{man} = \frac{g H_m}{V_{u_2} u_2}$$

$$V_{u_2} = \frac{g H_m}{\eta_{man} \times u_2} = \frac{9.81 \times 15}{0.75 \times 16.75}$$

$$= 11.71\,m/s$$

$$\tan \beta_2 = \frac{V_{f_2}}{u_2 - V_{u_2}}$$

$$\tan 40 = \frac{V_{f_2}}{16.75 - 11.71}$$

①

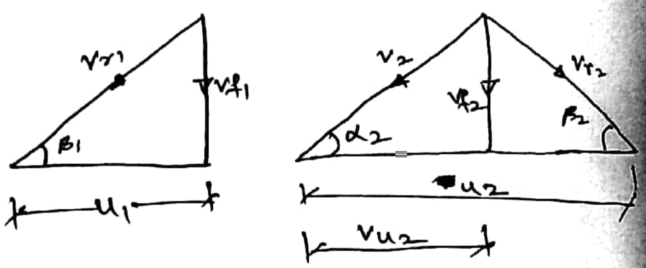
$$V_{f_2} = 4.23\,m/s$$

② $$V_2 = \sqrt{V_{f_2}^2 + V_{u_2}^2}$$

$$= \sqrt{4.23^2 + 11.71^2}$$

$$= 12.45\,m/s$$

③ α_2 is the angle made by the absolute velocity with direction of motion at outlet



$$\tan \alpha_2 = \frac{V_{f_2}}{V_{u_2}} = \frac{4.23}{11.71}$$

$$\alpha_2 = 19.86^\circ$$

④ Discharge Q

$$Q = A \times V_f$$

$$= \pi D_2 B_2 \times V_{f_2}$$

$$= \pi \times 0.4 \times 0.05 \times 4.23$$

$$= 0.265\,m^3/s.$$