Module-4

COMMUNICATION THROUGH BAND LIMITED CHANNELS

→ Most frequently encountered band limited channels are telephone channels, microwave line-of-sight radio channels, satellite channels & underwater acoustic channel.

→ A transmission channel with finite bandwidth is called bandlimited channels.

Digital transmission through bandlimited channels:

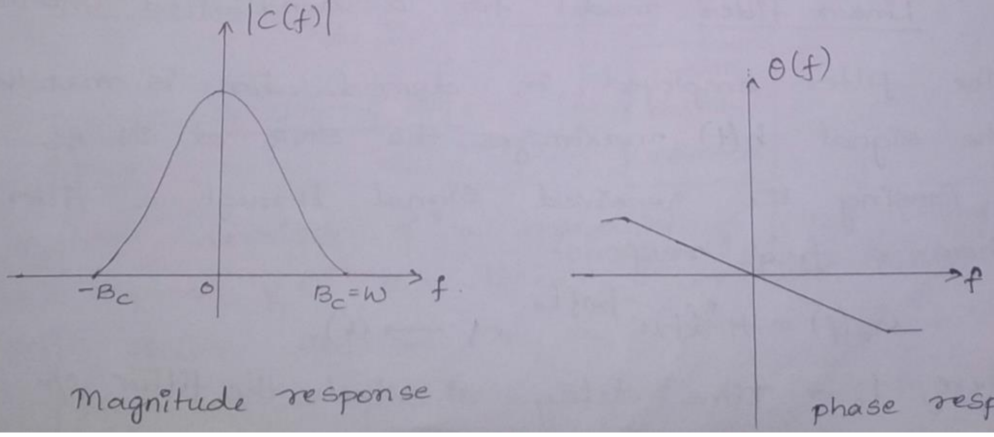
→ A bandlimited channel such as a telephone wireline is characterized as a linear filter with impulse response $c(t)$ & freq. response $C(f)$, where

$$C(f) = \int_{-\infty}^{\infty} c(t)\, e^{-j2\pi f t}\, dt \longrightarrow (1)$$

→ If the channel is a baseband channel that is bandlimited to 'B_c' Hz, then "$C(f)=0$" for $|f| > B_c$.

→ Any freq. component at the i/p of the channel that are higher than 'B_c' Hz will not be passed by the channel.

→ ∴ The magnitude & phase response of bandlimited channel is as shown below,



Magnitude response

phase response

Suppose the i/p. to a bandlimited channel is a signal waveform $g_T(t)$, where 'T' denotes that the signal is the o/p of Txr. Then the response of the channel is the convolution of $g_T(t)$ with c(t)

$$\therefore h(t) = \int_{-\infty}^{\infty} c(\tau)\, g(t-\tau)\, d\tau = c(t) * g_T(t) \longrightarrow (2)$$

In freq. domain,

$$H(f) = C(f) \cdot G_T(f). \longrightarrow (3)$$

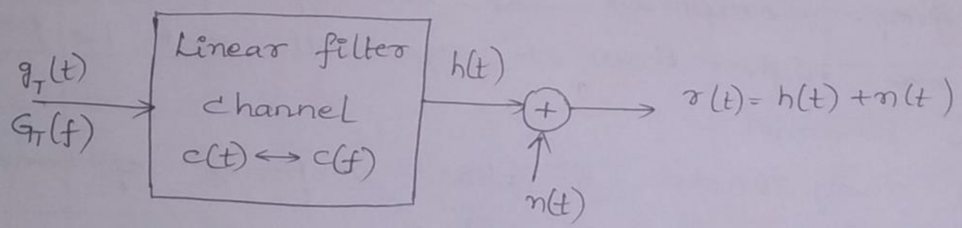
where, $G_T(f) \rightarrow$ spectrum of $g_T(t)$

$H(f) \rightarrow$ spectrum of $h(t)$.

When this signal is transmitted through channel, it gets distorted due to addition of AWGN

∴ The received signal is combination of transmitted signal h(t) & AWGN n(t).

∴ The linear filter channel model is as shown,



Linear filter model for a bandlimited channel.

The filter employed in demodulator is matched to the signal h(t) maximizes the SNR at its o/p.

∴ Passing the received signal through a filter that has a freq. response,

$$G_R(f) = H^*(f)\, e^{-j2\pi f t_0} \longrightarrow (4)$$

where, $t_0 \rightarrow$ time delay at which the filter o/p is sampled

→ The signal component at the o/p of the matched filter at the sampling instant $t=t_0$ is,

$$y_s(t_0) = \int_{-\infty}^{\infty} |H(f)|^2 df = \mathcal{E}_h$$ which is the energy in the

Channel o/p waveform $h(t)$.

→ The noise component at the o/p of matched filter has zero mean & a PSD $S_n(f) = \frac{N_0}{2}|H(f)|^2 \longrightarrow (6)$

Hence the noise power at the o/p of the matched filter has a variance,

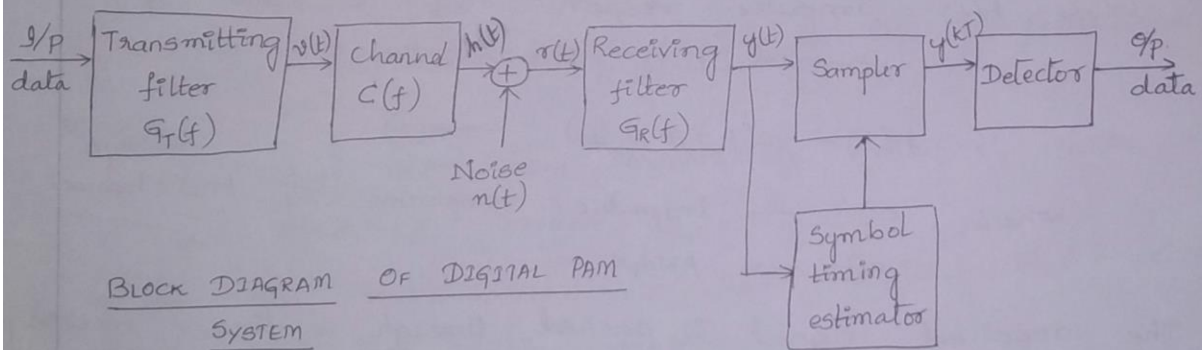
$$\sigma_n^2 = \int_{-\infty}^{\infty} S_n(f)\,df = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df = \frac{N_0\,\mathcal{E}_h}{2} \longrightarrow (7)$$

→ ∴ The SNR at the o/p of matched filter is,

$$\left(\frac{S}{N_0}\right)_0 = \frac{\mathcal{E}_h^2}{N_0\mathcal{E}_h/2} = \frac{2\,\mathcal{E}_h}{N_0}$$

* **Digital PAM transmission through bandlimited baseband channels :-**

→ Consider the baseband PAM communication system as shown in fig below,



BLOCK DIAGRAM OF DIGITAL PAM SYSTEM

→ The system consists of a transmitting filter having an impulse response $g_T(t)$, the linear filter chn with AWGN, a receiving filter with an impulse response $g_R(t)$, a sampler that periodically samples the o/p of the receiving filter &

a symbol detector

→ The sampler requires the extraction of a timing signal from the received signal. This timing signal serves as a clock that specifies the appropriate time instants for sampling the o/p of the receiving filter

→ In digital PAM system, the i/p binary data sequence is sub-divided into k-bit symbols & each symbol is mapped into a corresponding amplitude level that amplitude modulates the o/p of the transmitting filter

→ ∴ The baseband signal at the o/p of transmitting filter is expressed as,

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g_T(t-nT) \longrightarrow (1)$$

Where, $T = k/R_b$ is the symbol interval

R_b = bit rate

a_n = sequence of amplitude levels corresponding to the sequence of k-bit blocks of information bits

→ The channel o/p. at the receiver end is expressed as,

$$r(t) = \sum_{n=-\infty}^{\infty} a_n h(t-nT) + n(t) \longrightarrow (2)$$

where, $h(t)$ = impulse response of the ~~cade~~ sca of the transmitting filter & the channel

$$\therefore \; h(t) = c(t) * g_T(t) \longrightarrow (3)$$

where, $c(t) \longrightarrow$ impulse response of the channel.

$n(t) \longrightarrow$ AWGN

→ The received signal is passed through a linear receiving filter with the impulse response $g_R(t)$ & frequency response $G_R(f)$

→ If $g_R(t)$ is matched to $h(t)$, then its o/p SNR is maxim at the proper sampling instant.

→ ∴ The o/p of receiving filter is,

$$y(t) = \sum_{n=-\infty}^{\infty} a_n x(t-nT) + w(t) \longrightarrow (4)$$

where, $x(t) = h(t) * g_R(t) = g_T(t) * c(t) * g_R(t)$ & $w(t) = n(t) * g_R(t)$ denotes the additive noise at the o/p of the receiving filter.

→ The information symbols 'a_n' is recovered by sampling the o/p of receiving filter periodically for every 'T' seconds $t = mT$

∴ The sampled o/p is,

$$y(mT) = \sum_{n=-\infty}^{\infty} a_n x(mT - nT) + w(mT) \longrightarrow (5)$$

$$\left.\begin{aligned} \text{(or)} \quad y_m &= \sum_{n=-\infty}^{\infty} a_n x_{m-n} + w_m \\ &= x_0 a_m + \underbrace{\boxed{\sum_{n \neq m} a_n x_{m-n}}}_{\text{ISI}} + \underbrace{\boxed{w_m}}_{\text{Noise}} \end{aligned}\right\} \longrightarrow (6)$$

where, $x_m = x(mT)$, $w_m = w(mT)$ & $m = 0, \pm 1, \pm 2 \ldots$

* The first term in eqn (6) is the desired symbol a_m, with scaling factor 'x_0'.
* The second term in eqn (6) represents the effect of other symbols at the sampling instant $t = mT$, called the "INTERSYMBOL INTERFERENCE" [ISI]

→ In general, ISI causes degradation in the performance of DC system

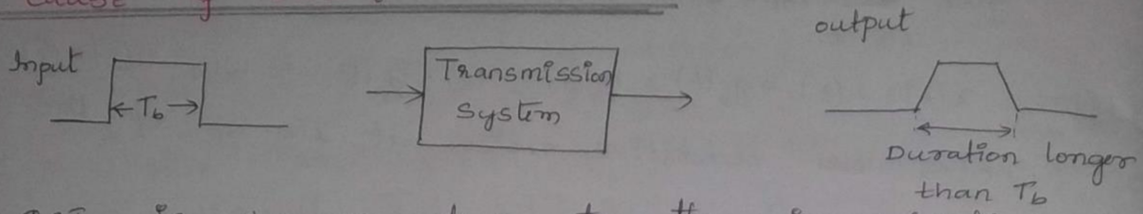
* The third term w_m, represents additive noise with zero mean & variance, $\sigma_w^2 = \frac{N_0 E_h}{2}$

→ When the receiving filter is matched to received signal $h(t)$, the scaling factor is,

$$x_0 = \int_{-\infty}^{\infty} h^2(t)\,dt = \int_{-\infty}^{\infty} |H(f)|^2 df = \int_{-W}^{W} |G_T(f)|^2 |C(f)|^2 df = E_h \longrightarrow (7)$$

→ ∴ By proper designing of transmitting & receiving filter the condition $x_n = 0$ for $n \neq 0$ can be satisfied, so that ISI is eliminated.

* Cause of ISI :-



→ ISI is causes due to the imperfections in the overall freq response of the system.

→ When a pulse of duration 'T_b' is transmitted through a bandlimited s/m, then the freq components in the i/p pulses are differentially attenuated & delayed by the s/m.

→ Due to this, the o/p pulse appears to be dispersed over an interval longer than 'T_b' sec

→ Due to this dispersion, the symbols will interfere with each other when transmitted over the communication channel, this results in "ISI"

* Effects of ISI :-

→ ISI introduces errors in the received signal which make decision difficult. Hence, the receiver can make error in deciding whether the received bit is '1' or '0'

* Remedy

→ The function which produces a zero ISI is a "sinc function" hence, instead of a rectangular pulse if we transmit a sinc pulse then ISI will be zero. This is known as Nyquist pulse shaping.

* **Signal design for bandlimited channels:-**

a) With zero ISI :-

→ Since $H(f) = C(f)\,G_T(f)$, the condition for distortion-free transmission is that freq. response characteristics '$C(f)$' of the channel must have a constant magnitude & a linear phase over the bandwidth of the transmitted signal

i.e
$$C(f) = \begin{cases} C_0\, e^{-j2\pi f t_0} & |f| \leq W \\ 0 & |f| > W \end{cases} \quad \rightarrow (1)$$

where,
$W \rightarrow$ channel bandwidth
$t_0 \rightarrow$ arbitrary finite delay [It is set to zero]
$C_0 \rightarrow$ constant gain factor [It is set to unity]

→ Under the condition that the channel is distortion-free & the bandwidth of $g_T(t)$ is 'W', then

$H(f) = G_T(f)\, C_0\, e^{-j\pi f t_0} = G_T(f) \cdot 1 \cdot e^0$

$$H(f) = G_T(f)$$

→ Consequently, the matched filter at the receiver has a frequency response $G_R(f) = G_T^*(f)$ & its o/p at the periodic sampling time $t = mT$ is,

$$y(mT) = x(0)\, a_m + \sum_{n \neq m} a_n x(mT - nT) + w(mT) \quad \rightarrow (2)$$

(or) $$y_m = x_0 a_m + \sum_{n \neq m} a_m x_{m-n} + w_m \quad \rightarrow (3)$$

→ ∴ The first term represents desired symbol 'a_m' with scaling factor 'x_0'

→ The second term represents ISI. The amount of ISI & noise that is present in the received signal can be viewed on an oscilloscope

→ The received signal is display on the vertical i/p with the horizontal sweep rate set at 1/T. The resulting oscilloscope display is called 'EYE PATTERN' because it resembles the human eye.

→ The interior region of the eye pattern is called the eye opening.

→ 1. The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI.

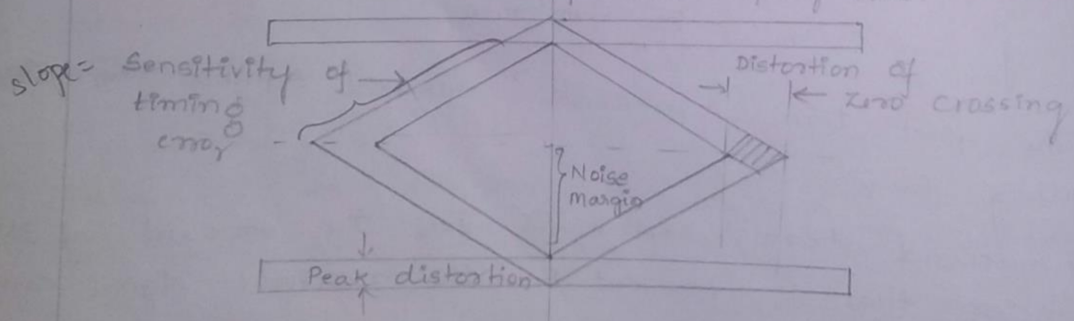
→ The preferred time for sampling is the instant at which the eye is open widest.

→ 2. The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling interval/time is varied.

→ 3. The height of eye opening, at a specified sampling time defines the margin over noise.

→ When the effect of ISI is severe, traces from the upper portion of eye pattern cross the traces from lower portion, with the result that the eye is completely closed.

→ In such cases, it is difficult to avoid errors due to combined effect of ISI & noise in the s/m.



Effect of ISI on eye opening

Q Consider a binary PAM s/m that transmits data at a rate of '1/T' bits/sec through an ideal channel of bandwidth 'W'. The sampled o/p from the matched filter at the rx^r is,

$y_m = a_m + 0.2a_{m-1} - 0.3a_{m-2} + w_m$,

where $a_m = \pm 1$ with equal probability. Determine the peak value of the ISI & the noise margin

→ Soln:

$a_{m-2} = a_{m-1}$

W.K.T.

$$y_m = x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + w_m \rightarrow ①$$

given, $y_m = a_m + 0.2\,a_{m-1} - 0.3a_{m-2} + w_m \rightarrow ①'$

$y_m = a_m + 0.2a_{m-1} + 0.3a_{m-1} + w_m$

Compare ① & ①',

$x_0 = 1$, $x_1 = 0.5$

$= \underbrace{+1 a_m}_{DO} + \underbrace{0.5a_{m-1}}_{ISI} + \underbrace{w_m}_{N}$

$x_0 = 1$, $x_1 = 0.2$ & $x_2 = -0.3$

The peak value of ISI occurs when $a_{m-1} = -a_{m-2}$,

So that the ISI term will take the peak value of 0.5

→ Since $x_0 = 1$ & $a_m = \pm 1$, the ISI causes a 50% reduction in the eye opening at the sampling times $t = mT$, $m = 0, \pm 1, \pm 2 \ldots$. Hence noise margin is reduced by 50% to a value of 0.5.

* ## Design of bandlimited signals for zero ISI - The Nyquist criterion:-

→ Consider a DCS which transmits the signal through an ideal bandlimited channel, when bandwidth of $g_T(t)$ is less than or equal to 'W'

→ ∴ the BW of the channel, which is the FT of the signal at the o/p of the receiving filter is,

$$X(f) = G_T(f) \cdot C(f) \cdot G_R(f) \longrightarrow \begin{cases} x(t) = h(t) * g_R(t) \\ \text{but, } h(t) = c(t) * g_T(t) \\ \therefore x(t) = c(t) * g_T(t) * g_R(t) \end{cases}$$

$$\text{W.K.T,}\quad C(f) = \begin{cases} C_0\, e^{-j2\pi f t_0} & |f| \leq W \\ 0 & |f| > W \end{cases}$$

denotes frequency response of the channel.

$\therefore X(f) = G_T(f)\, G_R(f)\, C_0\, e^{-j2\pi f t_0}$

→ For convenience, we set $C_0 = 1$ & $t_0 = 0$

$X(f) = G_T(f)\, G_R(f)$

where, $G_T(f)$ & $G_R(f)$ denotes freq responses of transmi-tting & receiving filters.

→ The o/p of receiving filter is periodically sampled at $t = mT$, where $m = 0, \pm 1, \pm 2 \ldots$ yields the expression,

$$y_m = x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + w_m$$

→ 1st term : desired symbol

2nd term : ISI

3rd term : Noise.

→ To remove the effect of ISI, it is necessary that $x(mT - nT) = 0$ for $n \neq m$ & $x(0) \neq 0$. $\therefore$ we assume $x(0)$

→ $\therefore$ the overall communication s/m has to be designed such that,

$$x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

This condition is known as "Nyquist pulse-shaping criterion" (or) "Nyquist condition for zero ISI"

* **Nyquist condition for zero ISI :-**

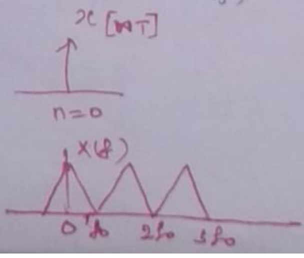
A necessary & sufficient condition for $x(t)$ to satisfy

$$x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \rightarrow (*)$$

, its F.T $X(f)$ must satisfy

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$$

$$X[f + mf_0] = T$$



Proof:- In general, $x(t) \xrightarrow{FT} X(f)$

$$\therefore \quad x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df \quad \rightarrow (1)$$

At sampling instants $t = nT$,

$$x(nT) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f nT}\, df \rightarrow (2)$$

(or)

$$x(nT) = \sum_{m=-\infty}^{\infty} \int_{\frac{(2m-1)}{2T}}^{\frac{(2m+1)}{2T}} X(f)\, e^{j2\pi f nT}\, df$$

$$x(nT) = \sum_{m=-\infty}^{\infty} \int_{-1/2T}^{1/2T} X\left(f + \frac{m}{T}\right) e^{j2\pi f nT}\, df.$$

$$= \int_{-1/2T}^{1/2T} \left[\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)\right] e^{j2\pi f nT}\, df.$$

$$\hookrightarrow = \int_{-1/2T}^{1/2T} Z(f)\, e^{j2\pi f nT}\, df \longrightarrow (3)$$

where,

$$Z(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) \longrightarrow (4)$$

$Z(f)$ is a periodic function with period $1/T$, $\therefore$ it can be expanded interms of its fourier series co-efficient as,

$$Z(f) = \sum_{n=-\infty}^{\infty} Z_n e^{j2\pi n f T} \longrightarrow (5)$$

where,

$$Z_n = T \int_{-1/2T}^{1/2T} Z(f)\, e^{-j2\pi f T n}\, df \longrightarrow (6)$$

comparing eq^n (3) & (6), $Z_n = T x(-nT) \longrightarrow (7)$

$\rightarrow$ To satisfy eq^n (*)

$$Z_n = \begin{cases} T & n = 0 \\ 0 & n \neq 0 \end{cases} \longrightarrow (8)$$

Subs this in eq^n (5)

$$\boxed{\begin{aligned} Z(f) &= T \quad \rightarrow (9) \\ \text{(or)} \quad \sum_{m=-\infty}^{\infty} X\left(f+\frac{m}{T}\right) &= T \quad \rightarrow (10) \end{aligned}}$$

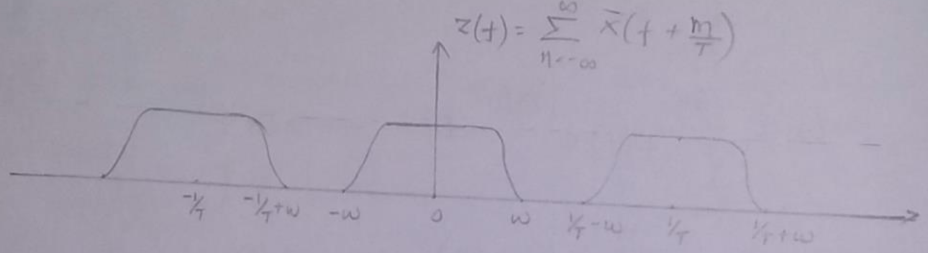
where, $T \rightarrow$ Symbol interval

→ Suppose that the channel has a bandwidth of W. Then $C(f)=0$ for $|f|>W$. consequently $X(f)=0$ for $|f|>W$

∴ The three cases are,

* Case 1 :- $T < \frac{1}{2W}$ (or) $\frac{1}{T} > 2W$; $f > 2W$

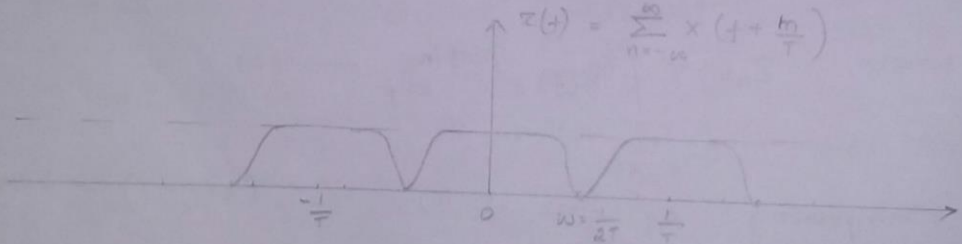
W.K.T. $z(f) = \sum_{n=-\infty}^{\infty} X\left(f+\frac{n}{T}\right)$ consists of non over lapping replicas of $X(f)$, which are separated by '$\frac{1}{T}$'.



Case 1 : $T < \frac{1}{2W}$

* Case 2 :- $T = \frac{1}{2W}$ (or) $2W = \frac{1}{T}$ {Nyquist rate} $f = 2W$

The replica of $X(f)$ is about to over lap as shown below & are separated by '$\frac{1}{T}$'



→ ∴ There exists only one $X(f)$ that results in $Z(f) = T$, namely,

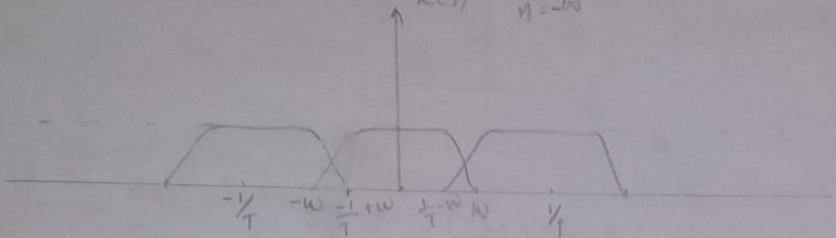
$$X(f) = \begin{cases} T & |f| < W \\ 0 & \text{otherwise} \end{cases}$$

(or) $X(f) = T\,\Pi\left(\frac{f}{2W}\right)$ which results in $x(t) = \text{Sinc}\left(\frac{t}{T}\right)$

* Case 3 :- $T > \frac{1}{2W}$, $Z(f)$ consists of overlapping replication of $X(f)$ separated by '$\frac{1}{T}$'

→ In case 3, there exists an infinite choices for $X(f)$, such that $Z(f) \equiv T$

$$Z(f) = \sum_{n=-\infty}^{\infty} X(f + n/T)$$



→ The pulse spectrum which satisfies case(3) ie $T > \frac{1}{2W}$ is raised cosine spectrum.

→ The raised cosine frequency characteristic is given as

$$X_{rc}(f) = \begin{cases} T & 0 \le |f| \le (1-\alpha)/2T \\ \frac{T}{2}\left[1 + \cos\left\{\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right\}\right] & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0 & |f| > \frac{1+\alpha}{2T} \end{cases}$$

where, $\alpha \longrightarrow$ roll-off factor, $0 \le \alpha \le 1$.

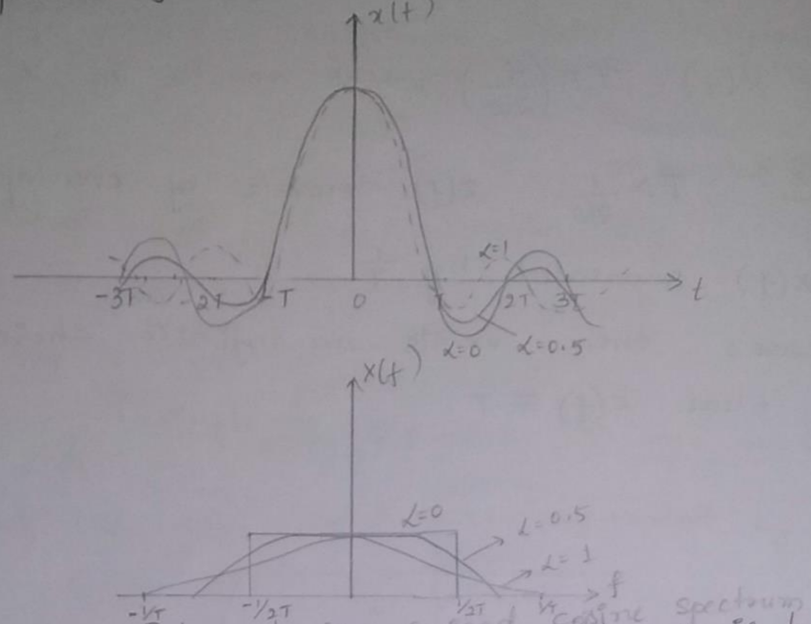
→ The bandwidth occupied by the signal beyond the Nyquist frequency '$\frac{1}{2T}$' is called the excess bandwidth & is usually expressed as a percentage of the Nyquist frequency.

→ The pulse $x(t)$ having the raised cosine spectrum is,

$$x(t) = \frac{\sin \pi t/T}{\pi t/T} \; \frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2} = \text{Sinc}(t/T)\,\frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2}$$

→ $x(t)$ is normalized so that $x(0) = 1$.

→ The raised cosine spectral characteristics & corresponding pulses for $\alpha = 0, 1/2, 1$ is as shown below,



Pulses having raised cosine spectrum & symbol rate is $\frac{1}{T} = 2W$.

→ When $\alpha = 0$, $x(t) = \mathrm{sinc}(t/T)$

→ When $\alpha = 1$, symbol rate is $\frac{1}{T} = W$, the tails of $x(t)$ decay as $1/t^3$ for $\alpha > 0$ &

→ In general, a mistiming error in sampling leads to a series of ISI components.

→ Due to smooth characteristics of the raised cosine spectrum, it is possible to design practical filters for the transmitter & receiver that approximate the overall desired frequency response.

→ In ideal channel with $C(f) = \Pi\left(\frac{f}{2W}\right)$, we have

$$X_{rc}(f) = G_T(f)\, G_R(f)$$

→ If the receiver filter is matched to the transmitter filter, $X_{rc}(f) = G_T(f)\, G_R(f) = |G_T(f)|^2$,

Ideally, $G_T(f) = \sqrt{|X_{rc}(f)|}\, e^{-j2\pi f t_0}$

Thus the overall raised cosine spectral characteristics is split evenly between the transmitting filter & receiving filter.

* Signal Design for bandlimited signals with controlled ISI - Partial response signals :- [correlative coding]

→ To realize practical transmitting & receiving filters, the symbol rate '1/T' should be reduced below the Nyquist rate of 2W symbols/sec.

→ To achieve a symbol transmission rate of 2W symbols/sec, the condition of zero ISI doesnot holds good. So, the required symbol transmission rate can be achieved by allowing for a controlled amount of ISI.

→ The condition of zero ISI is $x(nT)=0$ for $n \neq 0$. Suppose we design the bandlimited signal to have controlled ISI at one time instant ie. one nonzero value is added in the samples $x(nT)$.

→ The ISI introduced is controlled, hence it can be taken into account at the receiver.

$$\therefore \quad x(nT) = \begin{cases} 1 & n=0,1 \\ 0 & \text{otherwise} \end{cases} \longrightarrow (1)$$

W.K.T. the fourier series co-efficient 'z_n' is,

$$z_n = T\, x(-nT)$$

$$\therefore \quad z_n = \begin{cases} T & n=0,-1 \\ 0 & \text{otherwise} \end{cases} \longrightarrow (2)$$

We have, $$z(f) = \sum_{n=-\infty}^{\infty} z_n e^{j2\pi nfT}$$

Subs. $n=0,-1$

$$z(f) = z_0 e^{0} + z_{-1} e^{-j2\pi fT} = T + T e^{-j2\pi fT} \longrightarrow (3)$$

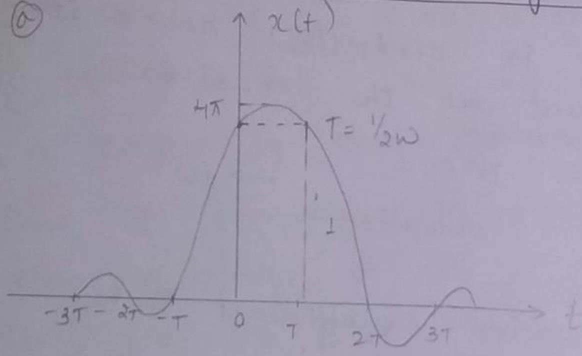
For $T = \frac{1}{2W}$,

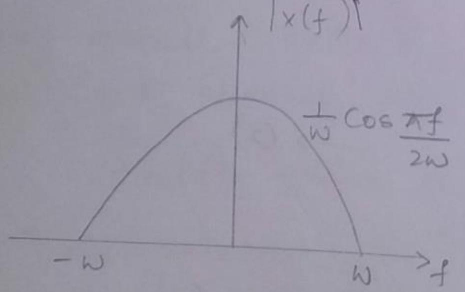
$$x(f) = \begin{cases} \frac{1}{2W}\left[1 + e^{-j\pi f/W}\right] & |f| < W \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{e^{-j\pi f/2W}}{2W}\left[e^{j\pi f/2W} + e^{-j\pi f/2W}\right] & |f| < W \\ 0 & \text{otherwise} \end{cases}$$

$$\hookrightarrow = \begin{cases} \frac{e^{-j\pi f/2W}}{W}\cos\left(\frac{\pi f}{2W}\right), & |f| < W \\ 0 & \text{otherwise.} \end{cases}$$

$\therefore$ $\boxed{x(t) = \text{sinc}(2Wt) + \text{sinc}(2Wt - 1)}$ This pulse is called a duobinary signal pulse.





→ From figure, the spectrum decays to zero smoothly, which means that physically realizable filters can be designed to approximate this spectrum & thus symbol rate of 2W is achieved.

Time & frequency domain characteristics of a duobinary signal

→ For duobinary signal pulse, $x(nT) = \begin{cases} 1 & \text{for } n = 0, 1 \\ 0 & \text{otherwise.} \end{cases}$

→ ∴ The samples at the o/p. of the receiving filter is,

$$y_m = b_m + w_m$$

where, $b_m = a_m + a_{m-1}$

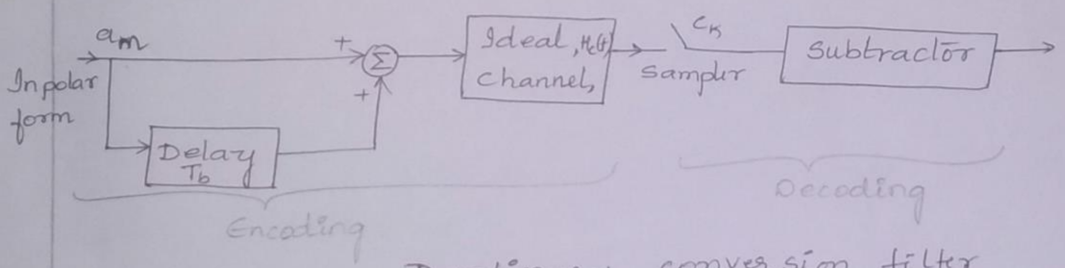
$$\therefore y_m = a_m + a_{m-1} + w_m$$

where, $a_m \rightarrow$ transmitted sequence amplitude.

$w_m \rightarrow$ sequence of additive Gaussian noise samples.

→ Neglect the noise introduced & consider $a_m = \pm 1$ with equal probability, them $b_m = -2, 0, 2$ with probabilities $1/4$, $1/2$, $1/4$ respectively.

→ If a_{m-1} is the detected symbol at interval $(m-1)$, its effect on 'b_m' can be eliminated by subtraction thus 'a_m' can be detected.



Duobinary conversion filter

* Adaptive equalizing filter [Generalized Correlative Coding :

→ To overcome the effect of ISI, a better way is to employ adaptive equalizing filters at the receiver end.

→ According to the attenuation occured for the delayed samples the filter is designed with corresponding inverse transfer functions using different co-efficients like $w_0, w_1, w_2 \dots$ & by summing all the components the actual signal is generated.

→ Equalization means, the process of correcting the channel induced distortion. It is adaptive as it adjust itself continuously during data transmission by operating on the i/p signal.

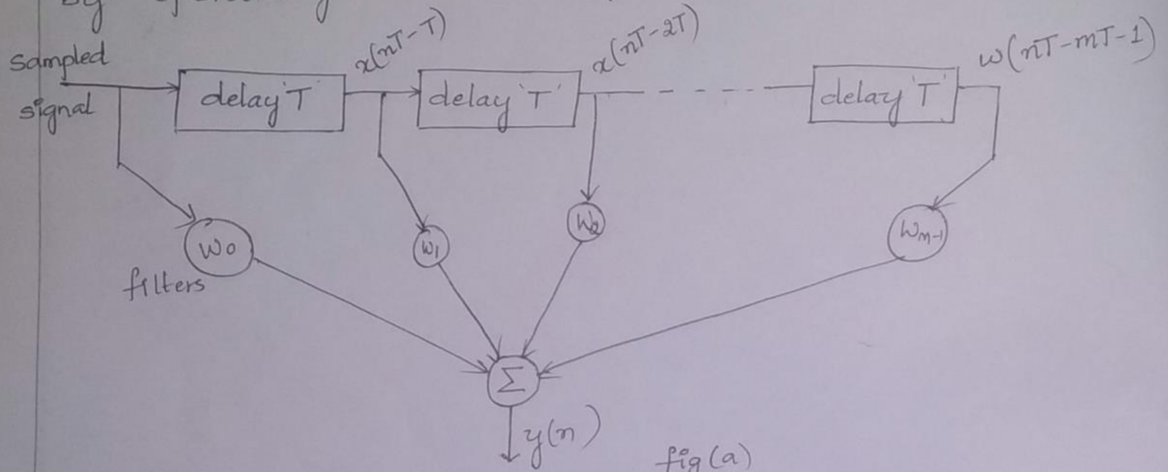


fig (a)

→ Adaptive channel equalizer is referred for channels whose characteristics change with time. In such case ISI change with time. Thus the channel equilizer must track such time variations in the channels & adopt its co-efficient to reduce the response ISI.

→ For practical implementation of equilizers, the optimum co-efficient vector is usually obtained by an iterative procedure.

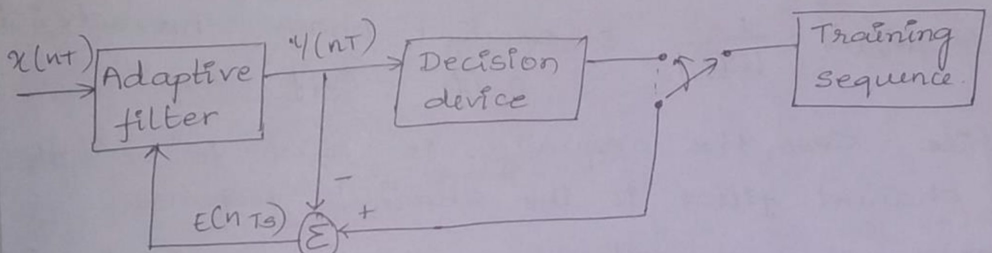
→ The adaptive equilizer consists of a tapped delay time filter with many taps as shown in figure (a).

whose co-efficients are updated in accordance with an algorithm & the adjustments are done in a step by step fashion synchronously with the incoming data.

There are two modes of operations in an adaptive equilizer during the training period, a known sequence is transmitted & a synchronized version of this signal is generated in the receiver where it is applied to the adaptive ~~filter~~ equilizer as the desired response.

When the training period is completed, the adaptive equalizer is switched to its second mode of operation which is decision directed mode.

In this mode, the error signal $E(nT) = b(nT) - Y(nT)$ is generated. Where, $Y(nT)$ is the equalizer o/p & $b(nT)$ is its final estimate of the transmitted signal, which makes the equilizer to operate satisfactory

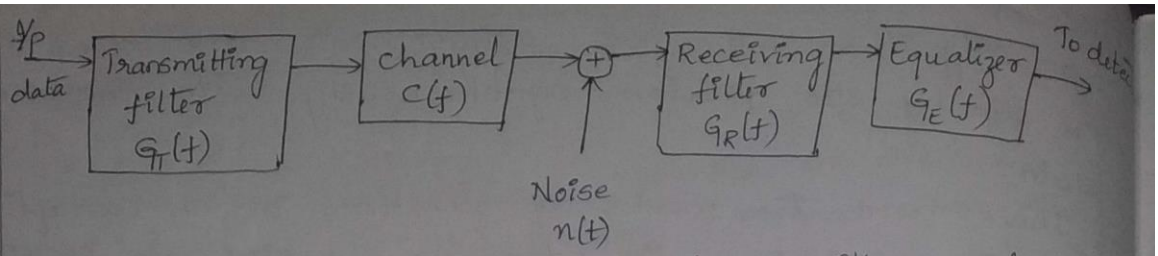


Adaptive equalizer

* Zero forcing equilizer [ZFE]:-

→ For channels whose frequency response characteristics are unknown & time variant, a linear filter with adjustable parameters, which is updated on a periodic basis to compensate for the channel distortion is employed.

→ Filters whose parameters are adjustable periodically is called an adaptive equalizer



Block diagram of a system with equalizer.

→ From above fig. it is clear that the demodulat consists of a receiving filter that has a frequency response $G_R(f)$ in cascade with equalizer.

→ Since $G_R(f)$ is matched to $G_T(f)$ & they are design so that their product satisfies $|G_R(f)||G_T(f)| = X_{rc}$, $|G_E(f)|$ compensate for channel distortion.

→ Hence the frequency response of equalizer must be equal to inverse of the channel response.

$$\therefore G_E(f) = \frac{1}{C(f)} = \frac{1}{|C(f)|} e^{-j\theta_c(f)}, \quad |f| \leq W.$$

Where, $|G_E(f)| = \frac{1}{|C(f)|}$ & equalizer phase characteristics $\theta_E(f) = -\theta_c(f)$

→ In this case, the equalizer is said to be the inverse channel filter to the channel response.

→ ∴ Inverse channel filter completely eliminates ISI caused by the channel response

→ Since the inverse channel filter (or) equalizer forces the ISI to be zero at the sampling times $t = nT$, the equalizer is called a zero forcing equalizer. Hence the i/p to the detector is,

$$y_m = a_m + w_m$$

where, w_m → noise component with $\mu = 0$ and

$$\text{Variance, } \sigma_w^2 = \int_{-\infty}^{\infty} S_n(f) |G_R(f)|^2 |G_E(f)|^2 df = \int_{-W}^{W} \frac{S_n(f) |X_{rc}(f)|}{|C(f)|^2} df$$

where, $S_n(f) \rightarrow$ power spectral density of noise

→ When the noise is white noise, $S_n(f) = \frac{N_0}{2}$ &

$$\therefore \text{Variance}, \sigma_w^2 = \frac{N_0}{2} \int_{-W}^{W} \frac{|X_{rc}(f)|}{|C(f)|^2} df$$

∴ The noise variance at the o/p of ZFE is higher than the noise variance at the o/p of the optimum receiving filter, $|G_R(f)|$.

* **Minimum mean square error [MMSE]:-**

→ The drawback of ZFE is it ignores the presence of additive noise

→ This alternative to overcome this is to relax the zero ISI condition & select the channel equalizer characteristic such that the combined power in the residual ISI & the additive noise at the o/p of the equalizer is minimized.

→ The above goal is achieved by using an channel equalizer that is optimized based on the minimum mean square error criterion.

→ Consider the noise corrupted o/p of FIR equalizer,

$$z(t) = \sum_{n=-N}^{N} c_n y(t - n\tau) \longrightarrow (1)$$

where, $y(t) \rightarrow$ i/p to equalizer

when o/p is sampled at $t = mT$,

$$\therefore z(mT) = \sum_{n=-N}^{N} c_n y(mT - n\tau) \longrightarrow (2)$$

→ The required response sample at the o/p of equalizer at $t = mT$ is the transmitted symbol 'a_m'.

→ The error is defined as the difference b/w a_m & $z(mT)$.

$z(mT)$ & the desired value 'a_m' is,

$$MSE = E\left[z(mT) - a_m\right]^2$$

$$= E\left[\sum_{n=-N}^{N} c_n\, y(mT - n\tau) - a_m\right]^2$$

$$\hookrightarrow = \sum_{n=-N}^{N}\sum_{k=-N}^{N} c_n c_k R_y(n-k) - 2\sum_{k=-N}^{N} c_k R_{AY}(k) + E(a_m^2) \rightarrow (3)$$

Where, $R_y(n-k) = E\left[y(mT - n\tau)\, y(mT - k\tau)\right]$

$$\& \; R_{AY}(k) = E\left[y(mT - k\tau)\, a_m\right] \rightarrow (4)$$

→ The solution for MMSE is obtained by differentiat-ing eqn. (3),

i.e. $\sum_{n=-N}^{N} c_n R_y(n-k) = R_{AY}(k) \quad , \quad k = 0, \pm 1, \ldots \pm N \rightarrow (5)$