Module 1: Bandpass signals to equivalent lowpass

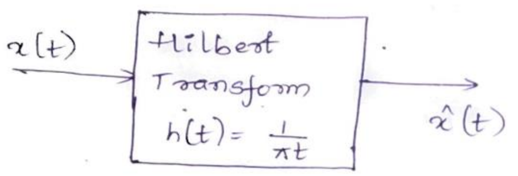
* Hilbert Transform :-

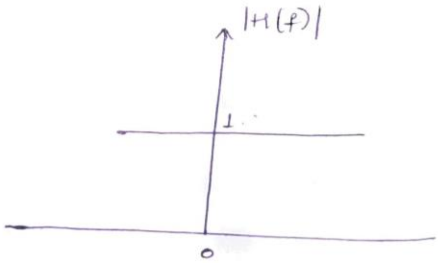
→ The fourier transform is useful for evaluating the frequency content of an energy signal

→ Signals can be separated based on phase selectivity also, which uses phase shifts between the pertinent signals to achieve the desired separation. [Phase shift of ±90°]

→ When the phase angles of all the components of a given signal are shifted by ±90°, the resulting function of time is known as Hilbert Transform of the signal.

→ Hilbert transform is also called "quadrature filter" because of its distinct property of providing a phase shift of ±90°.





Amplitude response

$\therefore \hat{x}(t) = x(t) * h(t)$

$\hat{x}(t) = x(t) * \frac{1}{\pi t}$

$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau}$$

→ H.T. is time domain to time domain transformation & is useful in phase selectivity.

→ H.T provides −90° phase shift for all +ve frequencies & +90° phase shift for all −ve frequencies

* Inverse Hilbert transform:-

We can recover the original signal $x(t)$ from $\hat{x}(t)$ by taking inverse hilbert transform as follows:

$$x(t) = \frac{-1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{t-\tau}\, d\tau$$

* Interpretation of Hilbert transform:-

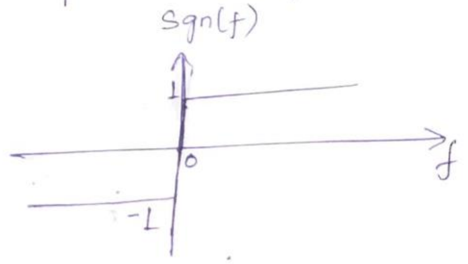
The fourier transform of $x(t)$ & $\frac{1}{\pi t}$ are;

$$x(t) \xrightarrow{F.T} X(f)$$

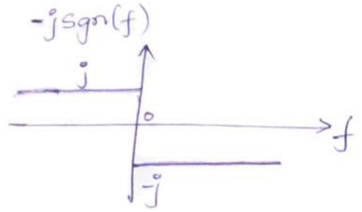
$$\frac{1}{\pi t} \xrightarrow{F.T} -j\,\mathrm{sgn}(f)$$

Where, Sgn is the signum function defined as,

$$\mathrm{sgn}(f) = \begin{cases} 1 & f > 0 \\ 0 & f = 0 \\ -1 & f < 0 \end{cases}$$

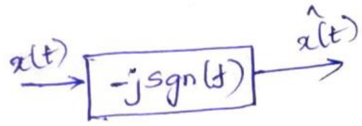


$$\therefore \hat{x}(t) = x(t) * \frac{1}{\pi t} \rightarrow (1)$$



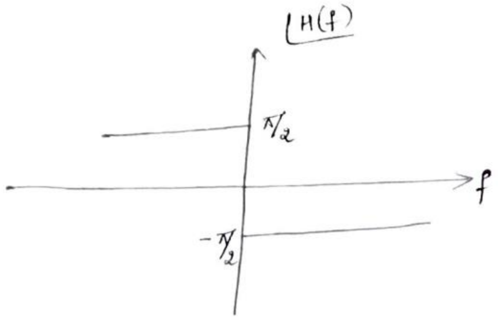
Take F.T

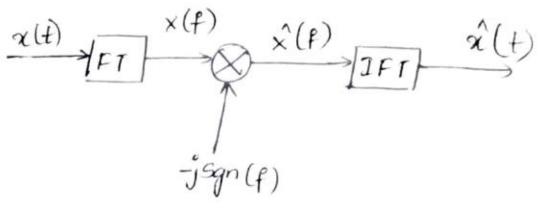
$$\hat{X}(f) = X(f)\left[-j\,\mathrm{sgn}(f)\right]$$



$$\hat{X}(f) = -j\,\mathrm{sgn}(f)\, X(f) \longrightarrow (2)$$

Thus the Hilbert transform $\hat{x}(t)$ of signal $x(t)$ is obtain by passing $x(t)$ through a linear two port devices whose transfer function is equal to $-j\,sgn(f)$.





* Properties of H.T. :-

Property 1 :- A signal $x(t)$ & its Hilbert transform $\hat{x}(t)$ have the same magnitude spectrum.

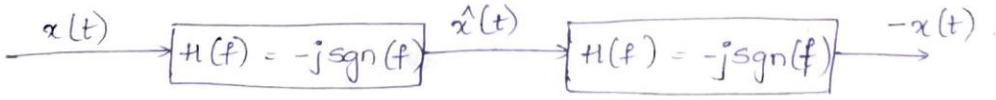
Proof :- F.T. of $\hat{x}(t)$, $\hat{X}(f) = -j\,sgn(f) \cdot X(f)$

$$|\hat{X}(f)| = |-j\,sgn(f)|\,|X(f)|$$

But $|-j\,sgn(f)| = 1$

$$\therefore \boxed{|\hat{X}(f)| = |X(f)|}$$

Property 2 :- If $\hat{x}(t)$ is the Hilbert transform of $x(t)$, then the Hilbert transform of $\hat{x}(t)$ is '$-x(t)$'.



Cascading of ideal two port devices to obtain the double H.T.

$\therefore H'(f) = H(f) \times H(f) = -j\,sgn(f) \times -j\,sgn(f)$

$\quad \hookrightarrow = j^2 sgn^2(f)$

But, $j^2 = -1$, $sgn^2(f) = 1$

$\therefore H'(f) = -1$ for all values of 'f'

Hence the FT of o/p is,

$X(f) \cdot H'(f) = -X(f)$

$-X(f) \xrightarrow{IFT} -x(t)$

Thus H.T of $\hat{x}(t)$ is $-x(t)$

Property 3: The signal $x(t)$ & its H.T $\hat{x}(t)$ are orthogonal functions for the entire time interval $(-\infty, \infty)$

i.e. $\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\,dt = 0$

Proof:-

W.K.T. $x(t) \xrightarrow{F.T} X(f)$

$\hat{x}(t) \xrightarrow{F.T} \hat{X}(-f)$

$\therefore \int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\,\hat{X}(-f)\,df$

W.K.T. $\hat{X}(f) = -j\,sgn(f)\,X(f)$

& $\hat{X}(-f) = -j\,sgn(-f)\,X(-f) = j\,sgn(f)\,X(-f)$

$$\therefore \int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} \cancel{j\,sgn(f)}\; X(f)\, j\, sgn(f)\, X(-f)\, df$$

$$\Rightarrow = j\int_{-\infty}^{\infty} X(f)\,X(-f)\, sgn(f)\, df$$

But, $X(f).\,X(-f) = |X(f)|^2$

$$\boxed{\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} j\, sgn(f)\, |X(f)|^2\, df}$$

From above eqn., it is product of odd & even function.

i.e The product of odd & even function is odd.

Where, $sgn(f) =$ odd function.

$|X(f)|^2 =$ even function

$\therefore$ The integration of an odd function over $-\infty$ to ∞ yields '0'

$$\therefore \boxed{\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0}$$

The additional properties of Hilbert transform are,

1. The magnitude spectra of a signal $x(t)$ & its Hilbert transform $\hat{x}(t)$ are identical
2. The H.T of an even function is odd & vice versa
3. The H.T of a real signal is also real

* Pre - Envelope :-

Pre-envelope is a complex signal generated by adding a signal with its Hilbert transform in quadrature.

The pre envelope of a signal is a complex function with either only the positive frequencies or only the negative frequencies. It is denoted as $x_+(t)$ & $x_-(t)$ & is defined as

$$x_+(t) = x(t) + j\hat{x}(t) \longrightarrow (1)$$

Where, $x(t)$ is the real part of the pre-envelope

$\hat{x}(t)$ is the imaginary part of the pre-envelope.

→ Let $X_+(f)$ represents the F.T of $x_+(t)$ & is given by,

$$X_+(f) = F[x(t) + j\hat{x}(t)]$$

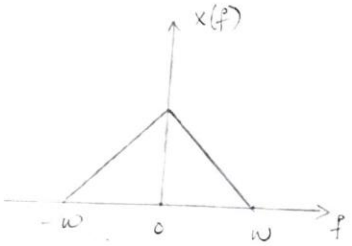
$$= X(f) + j\hat{X}(f) \longrightarrow (2)$$

W.K.T $\hat{X}(f) = -j\,\mathrm{sgn}(f)\,X(f)$

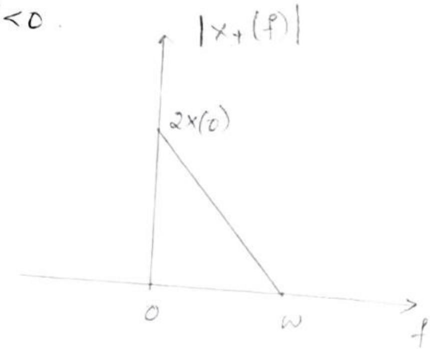
$$\therefore X_+(f) = X(f) - j^2\,\mathrm{sgn}(f)X(f) = X(f)\left[1 + \mathrm{sgn}(f)\right] \longrightarrow (3)$$

But, $\mathrm{Sgn}(f) = \begin{cases} 1 & f > 0 \\ 0 & f = 0 \\ -1 & f < 0 \end{cases}$ substituting in eq^n (3),

$$X_+(f) = \begin{cases} 2X(f) & \text{for } f>0 \\ X(0) & \text{for } f=0 \\ 0 & \text{for } f<0. \end{cases}$$



Amplitude spectrum of $x(t)$



Amplitude spectrum of pre-envelope $x_+(t)$

III|4 Pre-envelope for negative frequencies is defined as,

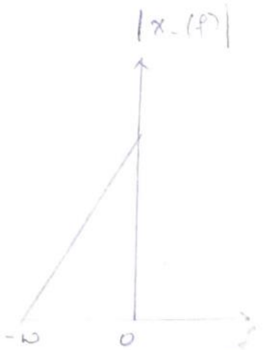
$$x_-(t) = x(t) - j\hat{x}(t)$$

The two pre-envelopes $x_+(t)$ & $x_-(t)$ are simply the complex conjugates of each other,

$$\text{ie} \quad x_-(t) = x_+^*(t)$$

$$\therefore X_-(f) = X(f) - j\hat{X}(f)$$
$$\hookrightarrow = X(f)\left[1 - \operatorname{sgn}(f)\right]$$

$$\therefore X_-(f) = \begin{cases} 0 & \text{for } f>0 \\ X(0) & \text{for } f=0 \\ 2X(f) & \text{for } f<0 \end{cases}$$



The two pre-envelopes are complex conjugate to each other, $x_+(t) = x_-^*(t)$

∴ The sum of $x_+(t)$ & $x_-(t)$ is,

$$x_+(t) + x_-(t) = [x(t) + j\hat{x}(t)] + [x(t) - j\hat{x}(t)]$$

$$= 2x(t)$$

$$\therefore \quad x(t) = \frac{x_+(t) + x_-(t)}{2}.$$

→ The spectrum of the pre-envelope $x_+(t)$ is nonzero only for positive frequencies; hence the use of a plus sign as the subscript.

→ The spectrum of the pre-envelope $x_-(t)$ is nonzero only for negative frequencies; hence the use of a minus sign as the subscript.

→ ∴ By appling the concept of pre-envelope to a band pass signal, the signal is ~~transformed~~ transformed into an equivalent low pass representation

* Complex envelopes :-

→ Let $s(t)$ be a bandpass signal, whose pre-envelope is expressed in the form,

$$s_+(t) = \tilde{s}(t)\exp(j2\pi f_c t) \longrightarrow (1).$$

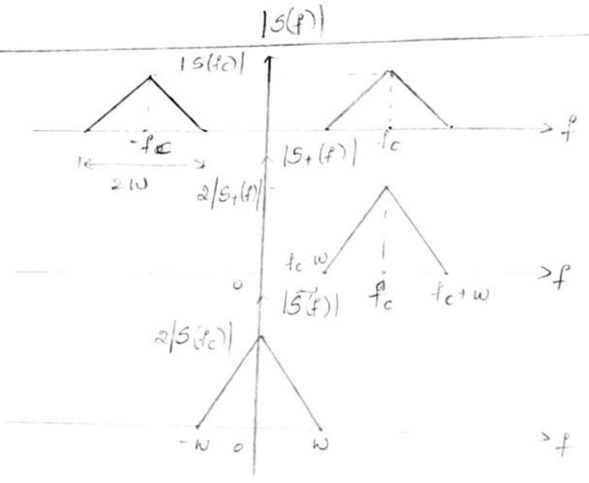
$$x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c}$$

where, $\tilde{s}(t)$ represents complex envelope of $s(t)$.

→ From fig. 1b. the spectrum of pre-envelope $s_+(t)$ is limited to the positive frequency band

$f_c - W \le f \le f_c + W.$

→ By applying the [fourier transform] frequency shift property of the F.T, the spectrum of complex envelope is limited to $-W \le f \le W$ & centered at $f = 0$.



(a) Magnitude spectrum of band-pass signal
(b) Magnitude spectrum of pre-envelope $s_+(t)$
(c) Magnitude spectrum of complex envelope $\tilde{s}(t)$

→ The complex envelope $\tilde{s}(t)$ of a band pass signal $s(t)$ is a complex low pass signal

* Canonical representation of Band Pass signals:-

A bandpass signal has a band of frequencies centered at 'f_c' with bandwidth 2W

→ The signal $x(t)$ is the real part of the pre-envelope $x_+(t)$. Hence the given band pass signal $x(t)$ can be expressed interms of complex envelope as,

$$x(t) = Re\left[\tilde{x}(t)\, e^{j2\pi f_c t}\right] \longrightarrow (1)$$

→ In general, $\tilde{x}(t)$ is a complex quantity which can be expressed as,

$$\tilde{x}(t) = x_I(t) + j x_Q(t) \longrightarrow (2)$$

Substituting (2) in (1) yields,

$$x(t) = Re\left\{\left[x_I(t) + j x_Q(t)\right] e^{j2\pi f_c t}\right\} \longrightarrow (3)$$

WKT, $e^{j\theta} = \cos\theta + j\sin\theta$

Here, $\theta = 2\pi f_c t$

$$\therefore e^{j2\pi f_c t} = \cos 2\pi f_c t + j\sin 2\pi f_c t \longrightarrow (4)$$

Subs (4) in (3), we get

$$x(t) = Re\left\{\left[x_I(t) + jx_Q(t)\right]\left[\cos 2\pi f_c t + j\sin 2\pi f_c t\right]\right\}$$

$$\therefore x(t) = Re\left\{x_I(t)\cos 2\pi f_c t + j\,x_I(t)\sin 2\pi f_c t + j\,x_Q(t)\cos 2\pi f_c t + j^2 x_Q(t)\sin 2\pi f_c t\right\}$$

$$\hookrightarrow = Re\left\{x_I(t)\cos 2\pi f_c t + j\,x_I(t)\sin 2\pi f_c t + jx_Q(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t\right\}$$

$$\therefore \boxed{x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t} \longrightarrow (5)$$

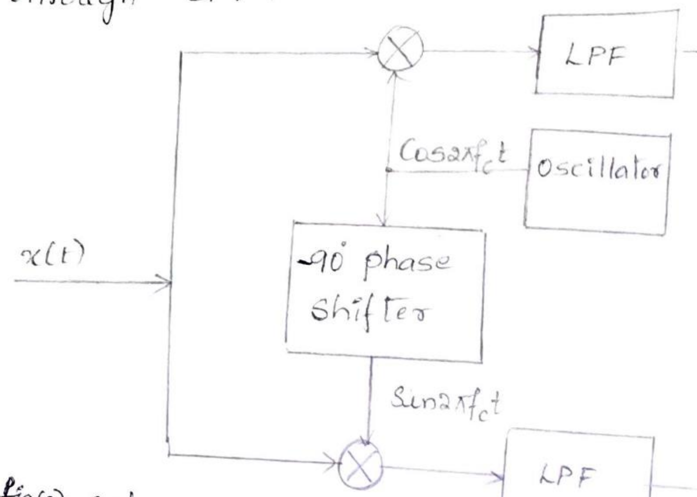
In above eqn., $x_I(t)$ is the inphase component of bandpass signal & $x_Q(t)$ is the quadrature component of bandpass signal $x(t)$

* Generation of In-phase & quadrature phase Components :- [~~Generation~~ & detection of bandpass signals]

→ The $x_I(t)$ & $x_Q(t)$ are low pass signals limited to the band $-W \le f \le W$.

→ The in-phase component $x_I(t)$ is produced by multiplying $x(t)$ with $\cos(2\pi f_c t)$ & passing it through a low pass filter.

→ The quadrature component $x_Q(t)$ is obtained by multiplying $x(t)$ with $\sin(2\pi f_c t)$ & passing it through LPF.



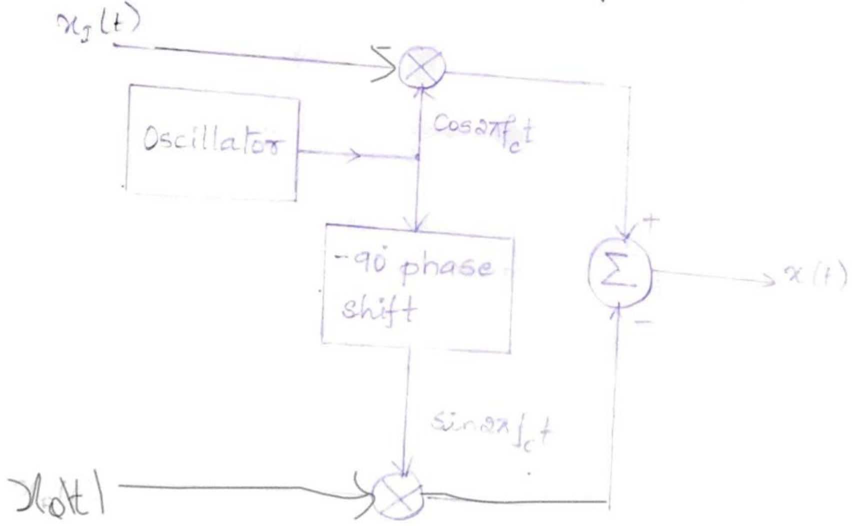
→ Fig(a) is referred as "analyzer" since it extracts $x_I(t)$ & $x_Q(t)$ from bandpass signal $x(t)$.

→ Fig(b) is referred as "synthesizer", as it reconstructs $x(t)$ from $x_I(t)$ & $x_Q(t)$.

fig(a): scheme for deriving the in-phase & quadrature components of bandpass signal

→ $x(t)$ is reconstructed by multiplying $x_I(t)$ & $x_Q(t)$ with $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$ respectively. & then the product is given to an adder.

→ Multiplication of $x_I(t)$ & $x_Q(t)$ with carrier is a linear modulation process.



Reconstruction of $x(t)$ from $x_I(t)$ & $x_Q(t)$

* Complex low pass representation of band pass systems :-

→ consider a narrow band signal $x(t)$, its FT is $X(f)$

→ let us assume the spectrum of $x(t)$ is limited to frequencies within $\pm W$ Hz of the carrier frequency 'f_c'. Let $W < f_c$

→ Let $x(t)$ be applied to linear time invariant band pass system with impulse response '$h(t)$' & frequency response '$H(f)$'

→ The frequency response is limited to frequencies within $\pm B$ of the carrier frequency 'f_c'

→ The system bandwidth is $2B$ which is narrower than the i/p signal bandwidth $2W$.

→ The bandpass impulse response can be expressed as,

$$h(t) = h_I(t) \cos 2\pi f_c t - h_Q(t) \sin 2\pi f_c t \longrightarrow (1)$$

→ The complex impulse response of a bandpass system is,

$$\tilde{h}(t) = h_I(t) + j h_Q(t) \longrightarrow (2)$$

Expressing $h(t)$ interms of $\tilde{h}(t)$

$$h(t) = \mathrm{Re}\left[\tilde{h}(t)\, e^{j2\pi f_c t}\right] \longrightarrow (3)$$

Where, $h_I(t)$, $h_Q(t)$ & $\tilde{h}(t)$ are all low pass functions limited to frequency band $-B \leq W \leq B$.

from eqn (3),

$$2h(t) = \tilde{h}(t)\, e^{j2\pi f_c t} + \tilde{h}^*(t)\, e^{-j2\pi f_c t} \longrightarrow (4)$$

Apply F.T

$$2H(f) = \tilde{H}(f-f_c) + \tilde{H}^*(f-f_c)$$

Where, $H(f) \rightleftharpoons h(t)$

$\tilde{H}(f) \rightleftharpoons \tilde{h}(t)$

& $H^*(f) = H(-f)$

$$\therefore \boxed{\tilde{H}(f-f_c) = 2H(f)} \quad \text{for } f>0 \longrightarrow (5)$$

$\tilde{H}(f)$ is given by,

$$\tilde{H}(f) = \tilde{H}_I(f) + j\tilde{H}_Q(f) \longrightarrow (6)$$

where, $\tilde{H}_I(f) = \frac{1}{2}\left[\tilde{H}(f) + \tilde{H}^*(-f)\right]$

$$\tilde{H}_Q(f) = \frac{1}{2j}\left[\tilde{H}(f) - j\tilde{H}^*(-f)\right]$$

$\therefore$ $\tilde{h}(t)$ is obtained by applying inverse F.T to $\tilde{H}(f)$

$$\therefore \boxed{\tilde{h}(t) = \int_{-\infty}^{\infty} \tilde{H}(f)\, e^{j2\pi f t}\, df} \longrightarrow (7)$$

from eqn (5), For a specified band pass freq response $H(f)$, we may determine the

* Line codes :-

Line code is a code chosen for use within a communication system for transmitting a digital signal over a transmission line.

Line coding represents the digital signal to be transmitted, with a waveform that is appropriate for the specific properties of the physical channel & of the receiving equipment.

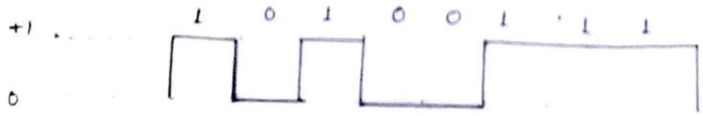
1 Unipolar format (or) ON-OFF signaling/ON-OFF keying:

* Symbol'1':- is represented by transmitting a pulse.
* Symbol'0':- is represented by switching off the pulse.

→ Disadvantage:- (NRZ) Waste of power due to the transmitted DC level.

(a) Unipolar NRZ signaling :-

When the pulse occupies the full duration of a symbol, then it is said to be non return to zero [NRZ] type.



Signals are represented as,

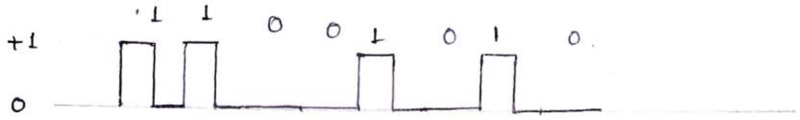
$$s(t) = \begin{cases} 1 & \text{for } 0 \le t \le T_b \text{ for symbol '1'} \\ 0 & \text{for } 0 \le t \le T_b \text{ for symbol '0'} \end{cases}$$

(b) Unipolar RZ Signaling :-

When the pulse occupies a fraction/one half of the symbol duration, it is RZ type

→ The special feature is the presence of delta function at $f = 0, \pm 1/T_b$ in the power spectrum of transmitted signal, it can be used for bit timing recovery at the receiver.

→ Disadvantage :- It requires 3dB more power than polar RZ.



→ $$s(t) = \begin{cases} 1, & 0 \le t \le T_b/2 \quad \text{for symbol '1'} \\ 0 & T_b/2 \le t \le T_b \quad \text{for symbol '1'} \\ 0 & 0 \le t \le T_b \quad \text{for symbol '0'} \end{cases}$$

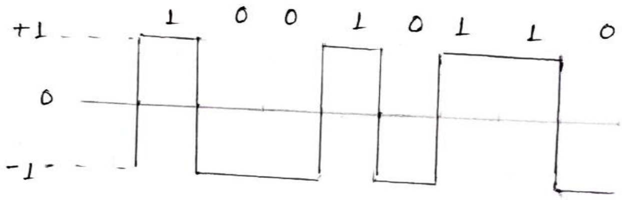
2. Polar Format :-

* Symbol '1' :- positive pulse is transmitted.
* Symbol '0' :- negative pulse is transmitted.

(a) Polar NRZ signaling :-

Disadvantage :- Power spectrum of the signal is large near zero frequency.

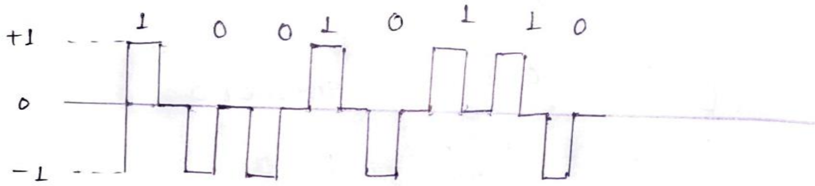
$$s(t) = \begin{cases} +1 & 0 \le t \le T_b \quad \text{for symbol '1'} \\ -1 & 0 \le t \le T_b \quad \text{for symbol '0'.} \end{cases}$$



* It has no (or) less DC component, provided that the 0's & 1's in the i/p data occur is equal proportion.
* This is efficient code becoz for a given code probability the code requires least transmission power.

(b) Polar RZ signaling :-

$$s(t) = \begin{cases} 1 & 0 \le t \le T_b/2 \text{ for symbol '1'} \\ 0 & T_b/2 \le t \le T_b \text{ for symbol 1} \\ -1 & 0 \le t \le T_b/2 \\ 0 & T_b/2 \le t \le T_b \end{cases} \text{ for symbol '0'}$$

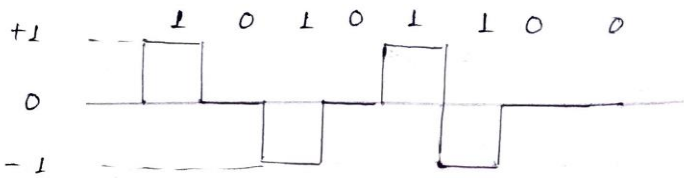


3. Bipolar Format or Pseudoternary signaling :-

In Bipolar format, positive & negative pulses are used alternatively for transmission of 1's & no pulse for transmission of 0's.

(a) Bipolar NRZ signaling :-

$$s(t) = \begin{cases} \pm 1 & 0 \le t \le T_b \text{ for symbol '1'} \\ 0 & 0 \le t \le T_b \text{ for symbol '0'} \end{cases}$$

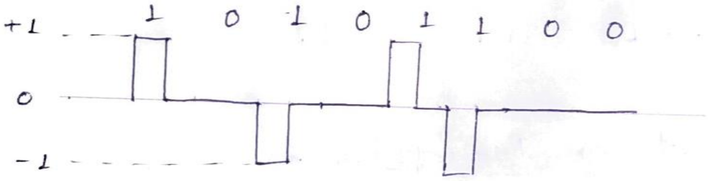


(b) Bipolar RZ signaling :-

→ The power spectrum of the transmitted signal has no DC component & relatively insignificant low-freq. components for the case when symbols '1' & '0' occur with equal probability.

→ Bipolar RZ signaling is also called Alternate mark inversion [AMI] signaling.

$$s(t) = \begin{cases} \left.\begin{array}{ll} \pm 1 & 0 \le t \le T_b/2 \\ 0 & T_b/2 \le t \le T_b \end{array}\right\} \text{symbol 1} \\ \\ \begin{array}{ll} 0 & 0 \le t \le T_b \end{array} \quad \text{symbol '0'} \end{cases}$$



* Power spectral densities of line codes :-

→ It is power per unit spectrum/bandwidth.

→ F.T. of autocorrelation gives PSD, which says how much power, DC components are present & required for a particular transmission.

→ PSD shows the energy distribution as a function of frequency.

→ Various signal formats like NRZ polar, NRZ unipolar etc can be considered as discrete amplitude modulated function & can be described interms of random process where,

$$x(t) = \sum_{K=-\infty}^{\infty} A_K v(t - kT) \longrightarrow (1)$$

Where, $A_K \longrightarrow$ Discrete random Variable

$v(t) \longrightarrow$ basic pulse shape ; $v(t) = rect\left(\frac{t}{T_b}\right)$

$T \longrightarrow$ symbol duration.

$v(t)$ is centered at origin i.e $t=0$ & normalized such that $v(0) = 1$.

To find power spectra of various line codes, Consider the source to be discrete, random & stationary. Autocorrelation of source is,

$$R_A(n) = E_l\left[A_K A_{K-n}\right] \longrightarrow (2)$$

$$\hookrightarrow = \sum_{i=1}^{l} [A_K\ A_{K-n}]_i\ P_i$$

Where, $E \longrightarrow$ expectation operator.

$l \rightarrow$ no of possible combinations

PSD is given by,

$$S_x(f) = \frac{1}{T_b} \left|V(f)\right|^2 \sum_{n=-\infty}^{\infty} R_A(n)\ e^{-j2\pi nfT} \longrightarrow (3)$$

Where, $V(f)$ is the F.T. of $v(t)$.

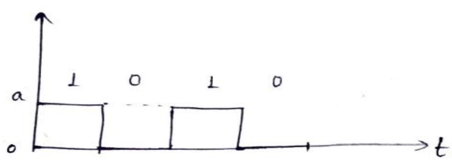
The value of $V(f)$ & $R_A(n)$ depends on the type of discrete PAM signal being considered.

1 NRZ Unipolar format :-

Suppose 0's & 1's of a random binary sequence occur with equal probability. For Unipolar NRZ format we have,

$$P(A_K = 0) = P(A_K = a) = \frac{1}{2}$$

$$\therefore A_K = \begin{cases} a & \text{for '1'} \\ 0 & \text{for '0'} \end{cases}$$



W.K.T. $R_A(n) = E\left[A_K\ A_{K-n}\right]$.

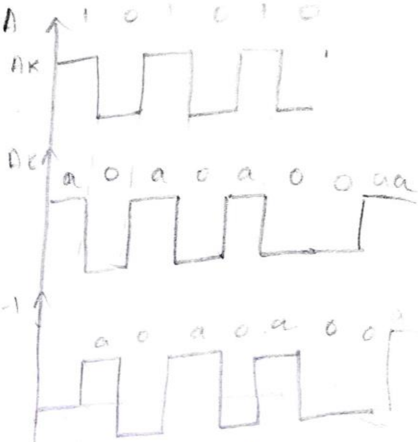
For $n=0$:

$$R_A(0) = E[A_K A_{K-0}] = E[A_K^2]$$

$$= 0^2 P(A_K=0) + a^2 P(A_K=a)$$

$$= 0 \times \frac{1}{2} + a^2 \times \frac{1}{2}$$

$$R_A(0) = \frac{a^2}{2}$$



* For $n \neq 0$: The probable combinations are 00, 0a, a0, aa. Assuming that the successive symbol in binary sequence are statistically independent, these four comb^{n} occur with probability '$\frac{1}{4}$'.

$$\therefore R_A(n) = E[A_K A_{K-n}] = \sum_{i=1}^{l} [A_K A_{K-n}]_i P_i$$

A_K	A_{K-n}	$A_K A_{K-n}$	P
0	0	0	1/4
0	a	0	1/4
a	0	0	1/4
a	a	a^2	1/4

$$R_A(n) = 0 \times \frac{1}{4} + 0 \times \frac{1}{4} + 0 \times \frac{1}{4} + a^2 \times \frac{1}{4}$$

$$\Rightarrow = \frac{a^2}{4}$$

$$\therefore R_A(n) = \begin{cases} \frac{a^2}{2} & n=0 \\ \frac{a^2}{4} & n \neq 0 \end{cases}$$

→ $V(f)$ is a F.T of $v(t)$, where $v(t)$ is a rectangular pulse of unit amplitude & pulse duration 'T_b', then the F.T. is,

$$V(f) = T_b \operatorname{sinc}(f T_b)$$

where, $\operatorname{sinc}\lambda = \frac{\sin \pi\lambda}{\pi\lambda}$; $\operatorname{sinc}(x) = \frac{\sin x}{x}$

Sub^s: $V(f)$ & $R_A(n)$ in (3)

$$S(f) = \frac{1}{T_b} \cdot T_b^2 \operatorname{sinc}^2(fT_b)\left[\sum_{n=0} \frac{a^2}{2} e^{-j2\pi f T_b n} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi f T_b n}\right]$$

$$S(f) = T_b \operatorname{sinc}^2(fT_b)\left[\frac{a^2}{2} \cdot e^0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi f T_b n}\right]$$

split $a^2/4$ $a^2/4$

$$S(f) = \frac{a^2 T_b \operatorname{sinc}^2(fT_b)}{4} + \frac{a^2 T_b \operatorname{sinc}^2(fT_b)}{4} + \frac{a^2 T_b \operatorname{sinc}^2(fT_b)}{4}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-j2\pi f T_b n} \rightarrow (4)$$

Take 2nd term & writing inside Σ, we get

$$\left[1 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty}\right]$$

$$S(f) = \frac{a^2 T_b \operatorname{sinc}^2(fT_b)}{4} + \sum_{n=-\infty}^{\infty} \frac{a^2 T_b \operatorname{sinc}^2(fT_b)}{4} e^{-j2\pi f T_b n} \rightarrow (5)$$

$$\sum_{n=-\infty}^{\infty}$$

W.K.T, poisson's formula is,

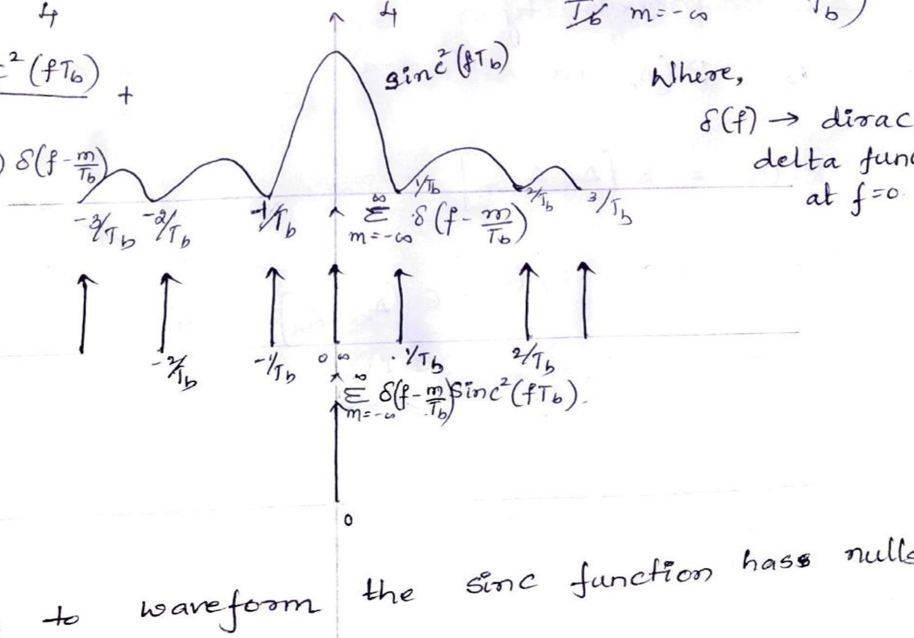
$$\sum_{n=-\infty}^{\infty} e^{-j2\pi n(fT_b)} = \frac{1}{T_b}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right) \longrightarrow (6)$$

Subs eqn. (6) in (5).

$$S(f) = \frac{a^2 T_b \operatorname{sinc}^2(fT_b)}{4} + \frac{a^2 \cancel{T_b} \operatorname{sinc}^2(fT_b)}{4} \cdot \frac{1}{\cancel{T_b}}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$

$$S(f) = \frac{a^2 T_b \operatorname{sinc}^2(fT_b)}{4} + \frac{a^2}{4}\sum_{m=-\infty}^{\infty} \operatorname{sinc}^2(fT_b)\, \delta\left(f - \frac{m}{T_b}\right)$$

Where, $\delta(f) \rightarrow$ dirac delta funcn. at $f=0$

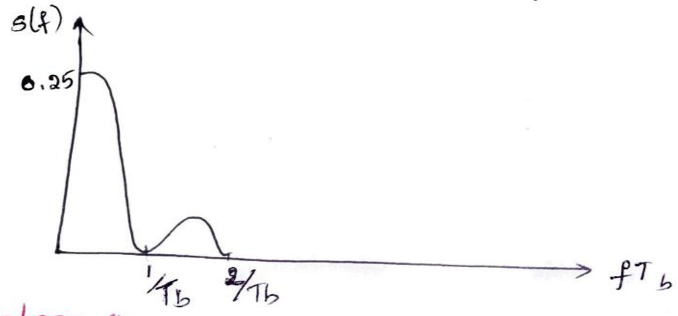


Refering to waveform the sinc function hass nulls at $\pm 1/T_b$ $\pm 2/T_b$ & $\pm 3/T_b$ ....

Multiplying the two signals at $m=0$

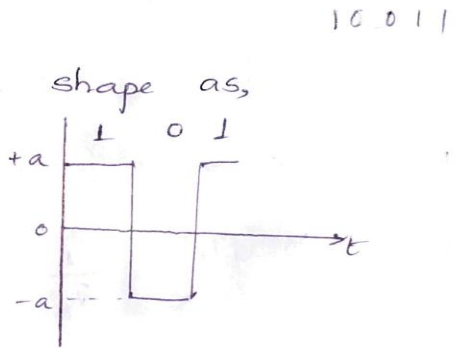
$$S(f) = \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) + \frac{a^2}{4}\delta(f).$$

∴ PSD curve is given by,



2. NRZ polar :-

The basic pulse takes the shape as,



A_K is given by,

$$A_K = \begin{cases} +a & \text{for } 1 \\ -a & \text{for } 0 \end{cases}$$

Assuming that symbol '1' & '0' occurs with equal probability

i.e. $P(A_K = +a) = P(A_K = -a) = \frac{1}{2}$

$$R_A(n) = E[A_K A_{K-n}]$$

∴ When $n=0$: $R_A(0) = E[A_K \cdot A_K] = E[A_K^2]$

$$R_A(0) = a^2 P(A_K = +a) + a^2 P(A_K = -a)$$

$$= \frac{a^2}{2} + \frac{a^2}{2}$$

$$\rightarrow = a^2$$

When $n \neq 0$:- the possible combinations are

$a\,a$, $a \times -a$, $-a \times a$, $-a \times -a$ with probabilities of '$\frac{1}{4}$' each

A_K	A_{K-n}	$A_K A_{K-n}$
a	a	a^2
a	$-a$	$-a^2$
$-a$	a	$-a^2$
$-a$	$-a$	a^2

$$R_A(n) = E[A_K \, A_{K-n}]$$

$$= \frac{a^2}{4} + \left(\frac{-a^2}{4}\right) + \left(\frac{-a^2}{4}\right) + \frac{a^2}{4}$$

$$= 0$$

$$\therefore R_A(n) = \begin{cases} a^2 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

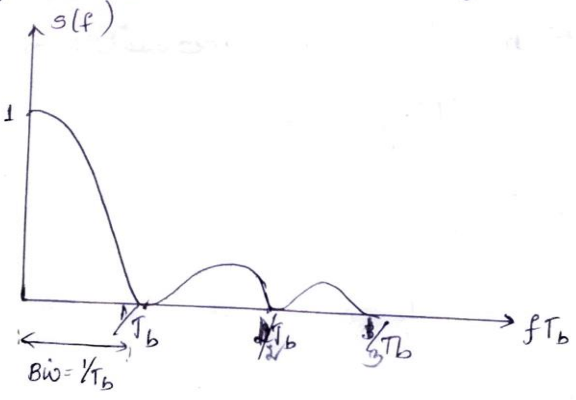
$\therefore$ W.K.T. $V(f) = T_b \operatorname{sinc}(fT_b)$

sub. $V(f)$ & $R_A(n)$ in (3),

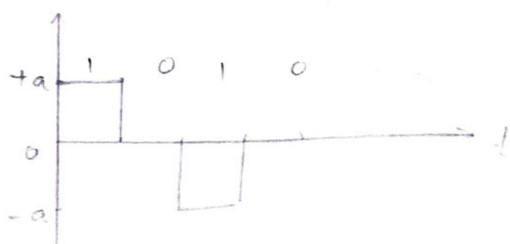
$$\therefore S(f) = \frac{1}{T_b} T_b^2 \operatorname{Sinc}^2(fT_b) \left[\sum_{n=0} a^2 e^{-j2\pi f n T_b}\right]$$

$$\boxed{S(f) = a^2 T_b \operatorname{sinc}^2(fT_b)}$$

$\therefore$ For NRZ polar format, normalized PSD is as shown below,



3 NRZ Bipolar format :-



It has 3 levels. ie. $+a$, 0 & $-a$.

$$A_K = \begin{cases} \pm a & \text{for } 1 \\ 0 & \text{for } 0 \end{cases}$$

Assuming 1's & 0's will occur with equal probability the probabilities of each levels are.

$P(A_K = 0) = \frac{1}{2}$; $P(A_K = 1) = \frac{1}{2}$ → $P(A_K = +a) = \frac{1}{4}$, $P(A_K = -a) = \frac{1}{4}$

W.K.T, $R_A(n) = E[A_K A_{K-n}]$

<u>When $n=0$:-</u> $R_A(0) = E[A_K A_K] = E[A_K^2]$

$$= 0 \times \frac{1}{2} + a^2 \times \frac{1}{4} + (-a)^2 \times \frac{1}{4}$$

$$\hookrightarrow = \frac{a^2}{2}$$

<u>When $n=1$:-</u>

$E[A_K A_{K-n}]$ has 4 probabilities ie 00, 01, 10, 11 with probability of $1/4$

A_K	A_{K-1}	$A_K A_{K-1}$	P
0	0	0	
0	$\pm a$	0	$1/4$
$\pm a$	0	0	
$\pm a$	$\mp a$	$-a^2$	

$\therefore R_A(1) = E[A_K A_{K-1}]$

$$= 0 \times \frac{1}{4} + 0 \times \frac{1}{4} + 0 \times \frac{1}{4} - a^2 \times \frac{1}{4}$$

$$\hookrightarrow = \frac{-a^2}{4}$$

$\therefore R_A(n) = R_A(-n)$

$\therefore R_A(1) = R_A(-1) = -\frac{a^2}{4}$

For $n>1$:

Eg $n=2$: $R_A(n) = E[A_k A_{k-2}]$

$$= 0\cdot\frac{1}{4} + 0\cdot\frac{1}{4} + 0\cdot\frac{1}{4} - \frac{a^2}{8} + \frac{a^2}{8}$$

$\hookrightarrow = 0$

$0 \;\pm a \rightarrow 0 \;] \rightarrow \frac{1}{4}$
$\pm a \;\; 0 \rightarrow 0$
$0 \;\; 0 \rightarrow 0$
$\pm a \pm a \rightarrow a^2 \} \rightarrow \frac{1}{4} < \frac{1}{8}$
$\mp a \pm a \rightarrow -a^2 \;\; \frac{1}{8}$

$$\therefore R_A(n) = \begin{cases} a^2/2 & \text{for } n=0 \\ -a^2/4 & \text{for } n=\pm 1 \\ 0 & \text{for } n>1 \end{cases}$$

$= \frac{1}{T_b}(V(f))^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi fn}$

WKT $V(f) = T_b \,\text{Sin}\, c(fT_b)$

Sub. $R_A(n)$ & $V(f)$ in (3).

$$S(f) = \frac{1}{T_b}\cdot T_b^2 \,\text{Sinc}^2(fT_b)\left[\sum_{n=-1} R_A(-1) e^{-j2\pi fnT_b} + \sum_{n=0} R_A(0) e^{-j2\pi fnT_b} + \sum_{n=1} R_A(1) e^{-j2\pi fnT_b}\right]$$

$$S(f) = T_b \,\text{Sinc}^2(fT_b)\left[\frac{-a^2}{4} e^{+j2\pi f(1)T_b} + \frac{a^2}{2}e^0 + \left(\frac{-a^2}{4}\right) e^{-j2\pi f(1)T_b}\right]$$

$$= T_b \,\text{Sinc}^2(fT_b)\left[\frac{a^2}{2} - \frac{a^2}{4}\left(e^{-j2\pi fT_b} + e^{j2\pi fT_b}\right)\right]$$

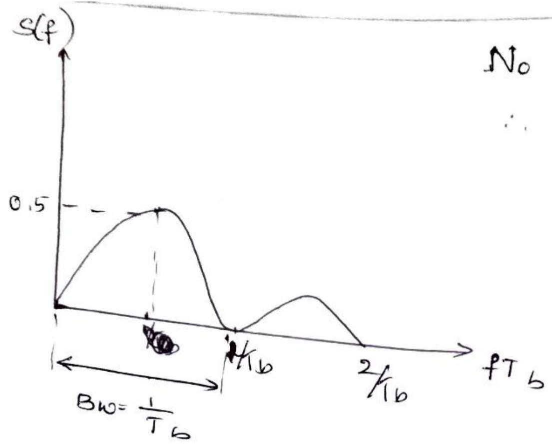
$\theta = 2\pi f T_b$

$$= T_b \,\text{Sinc}^2(fT_b)\left[\frac{a^2}{2} - \frac{a^2 \cos 2\pi fT_b}{2}\right]$$

$\frac{1-\cos 2\theta}{2} = \sin^2\theta$

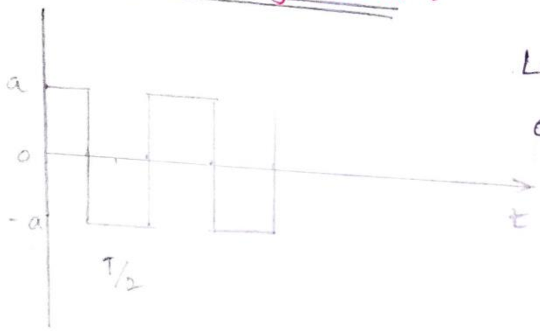
$$\hookrightarrow = \frac{T_b a^2}{2} \text{Sinc}^2(fT_b)\left[1 - \cos 2\pi fT_b\right] = \frac{T_b a^2}{2}\text{Sinc}^2(fT_b)\cdot 2\sin^2\pi fT_b$$

$$\therefore \boxed{S(f) = a^2 T_b \,\text{Sinc}^2(fT_b)\,\sin^2(\pi fT_b)}$$



No DC component is present
∴ frequency of DC component is zero.

4. Manchester format :-



Let us assume '0' & '1' occurs with equal probability

$$P(A_K = a) = \frac{1}{2}$$

$$P(A_K = -a) = \frac{1}{2}$$

WKT. $R_A(n) = E[A_K A_{K-n}]$

when n=0 :-

$$R_A(0) = E[A_K A_K] = E[A_K^2]$$

$$= \frac{a^2}{2} + \frac{a^2}{2} = a^2$$

when n≠0 :-

$$R_A(n) = \frac{a^2}{4} - \frac{a^2}{4} - \frac{a^2}{4} + \frac{a^2}{4} = 0$$

$$\therefore R_A(n) = \begin{cases} a^2 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

V(f) for manchester format is given by,

$$V(f) = \int_{-T_b}^{0} \pm e^{-j2\pi ft}\, dt + \int_{0}^{T_b/2} e^{-j2\pi ft}\, dt$$

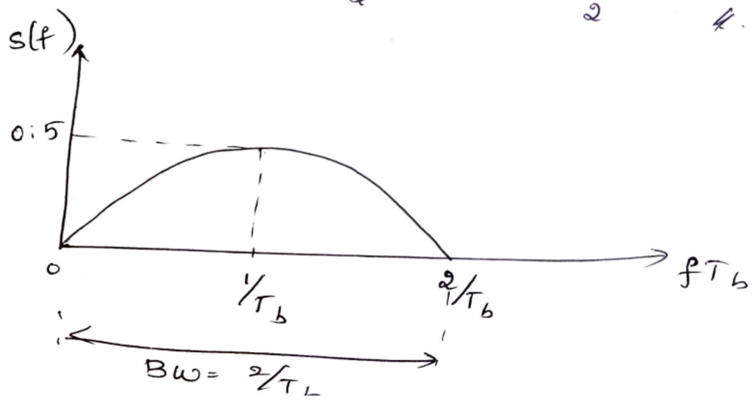
$$\therefore \quad V(f) = j T_b \, \text{Sinc} \frac{f T_b}{2} \, \text{Sin} \frac{\pi f T_b}{2}$$

$$|V(f)|^2 = T_b^2 \, \text{Sinc}^2 \frac{f T_b}{2} \, \text{Sin}^2 \frac{\pi f T_b}{2}$$

Subs. $|V(f)|^2$ & $R_A(n)$ in (3)

$$S(f) = \frac{1}{T_b} T_b^2 \, \text{Sinc}^2 \frac{f T_b}{2} \, \text{Sin}^2 \frac{\pi f T_b}{2} \left[\sum_{n=0} a^2 e^{-j 2\pi f n T_b} + 0 \right]$$

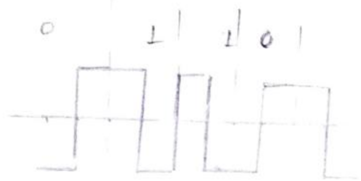
$$\hookrightarrow = T_b \, \text{Sin}^2 \frac{\pi f T_b}{2} \, \text{Sinc}^2 \frac{f T_b}{2} \, a^2 \quad //.$$



* Manchester format (or) Biphase Baseband Signaling :-

* Symbol '1' :- represented by transmitting a positive pulse for one-half of the symbol duration followed by a negative pulse for the remaining half of the symbol duration.

* Symbol '0' :- represented by transmitting a negative pulse for one-half of the symbol duration followed by a positive pulse for the remaining half of the symbol duration.



$$s_i(t) = \begin{cases} +1 & 0 \le t \le T_b/2 \\ -1 & T_b/2 \le t \le T_b \end{cases} \Bigg\} \text{ Symbol '1'}$$

$$\begin{cases} -1 & 0 \le t \le T_b/2 \\ +1 & T_b/2 \le t \le T_b \end{cases} \Bigg\} \text{ Symbol '0'}$$

→ It has no DC component. It offers better synchronization compared to other formats, but it requires double the bandwidth compared to other formats.

* Selection of encoding formats for a particular application :-

The parameters to be seen to choose a particular coding scheme are as follows,

1. DC component should be avoided to enable magnetic recording & transformer coupling.
2. The bandwidth should be as less as possible.
3. Self clocking mechanism.

Error detection & correction facility.

Transparency & ruggedness.

Power spectrum should match with frequency response of the channel.

Transmitted power should be as less as possible.

HDB3 coding

→ HDB3 encoding is same as AMI, except that a sequence of 4 consecutive 0's are encoding using 'Violation bit'

→ This bit will have same polarity as the last 1-bit which was sent using the AMI encoding.

→ The purpose of HDB3 is to prevent long runs of 0's in data stream & helps DPLL for tracking the centre of each bit. ∴ it is also called "run length limited" code

→ The use of violations in the signal gives extra 'edge' which makes synchronization possible & data retrieval will be more accurate.

→ An additional technique is used to stop DC voltage being introduced by having two many zero. This works by adding a balancing pulse to any pattern of more than 4 bits as zeros.

→ The value of 'B' is assigned as +ve (or) -ve, so as to make alternate "V" of opposite polarity.

→ Tabular column shows HDB3 encoding rules.

Transmitted data	HDB3 encoded pattern
0	0
1	AMI
0000	000V
0000 0000	B00V B00V

→ HDB3 coding of 0000_2

Parity of +/- bits since previous 'V'	Pattern	Previous pulse	Coded
odd	000V	+	000+
		−	000−
Even	B00V	+	−00−
		−	+00+

* B3ZS:-

→ At North American T3 rate, bipolar violations are inserted if 3 or more consecutive zeros occur. This line code is called "bipolar with three zero substitution".

→ Each run of 3 consecutive zeros is replaced by "OOV" or "BOV"

→ The choice is made to ensure that consecutive violations are of differing polarity i.e. separated by odd number of normal B marks.

→

No of bits since last V	Pattern	Polarity of last 'B'	Coded
odd	OOV	+	OO+
		-	OO-
Even	BOV	+	-O-
		-	+O+

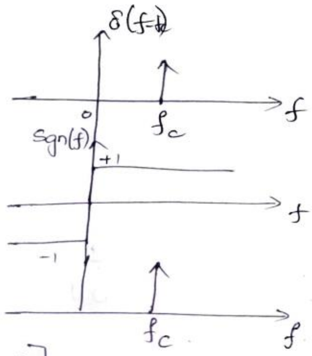
* **B6ZS :-**

→ At North America T2 rate, bipolar violations are inserted if 6 or more consecutive zeros occur. This line code is called bipolar with six-zero substitution & replaces 6 consecutive zeros with the pattern "OVBOVB".

* **Problems :-**

1. Find the H.T. of $x(t) = \cos 2\pi f_c t$

→ Soln :- $\cos 2\pi f_c t \rightarrow \boxed{90° \text{ PS}} \rightarrow \sin 2\pi f_c t$

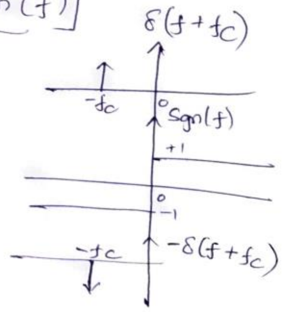


$$\hat{x}(f) = -j\,\text{sgn}(f)\, x(f).$$

$$x(f) = F.T.\left[\cos 2\pi f_c t\right] = \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$\therefore \hat{x}(f) = \frac{-j}{2}\,\text{sgn}(f)\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$= \frac{1}{2j}\left[\delta(f-f_c)\,\text{sgn}(f) + \underbrace{\delta(f+f_c)\,\text{sgn}(f)}_{-\delta(f+f_c)}\right]$$



$$\hookrightarrow = \frac{1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

I.F.T

$$\boxed{\hat{x}(t) = \sin 2\pi f_c t}$$

2. Find the H.T of $x(t) = \sin 2\pi f_c t$

→ Soln :- $\sin 2\pi f_c t \rightarrow \boxed{90° \text{ PS.}} \rightarrow -\cos 2\pi f_c t$

$$\hat{x}(f) = -j\,\text{sgn}(f)\, x(f)$$

$$x(f) = F.T\{\sin 2\pi f_c t\} = \frac{1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

$$\therefore \hat{x}(f) = \frac{-\cancel{j}\,\text{sgn}(f)}{2\cancel{j}}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

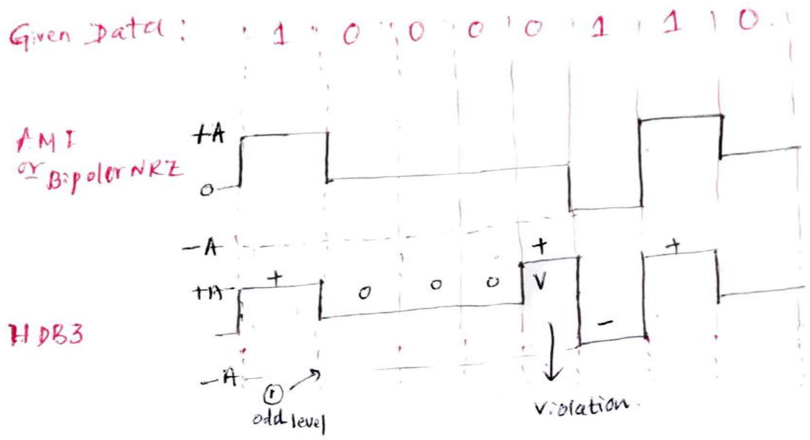
$$= \frac{-1}{2}\left[\delta(f-f_c)\,\text{sgn}(f) - \underbrace{\delta(f+f_c)\,\text{sgn} f}_{-\delta(f+f_c)}\right]$$

$$= -\frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

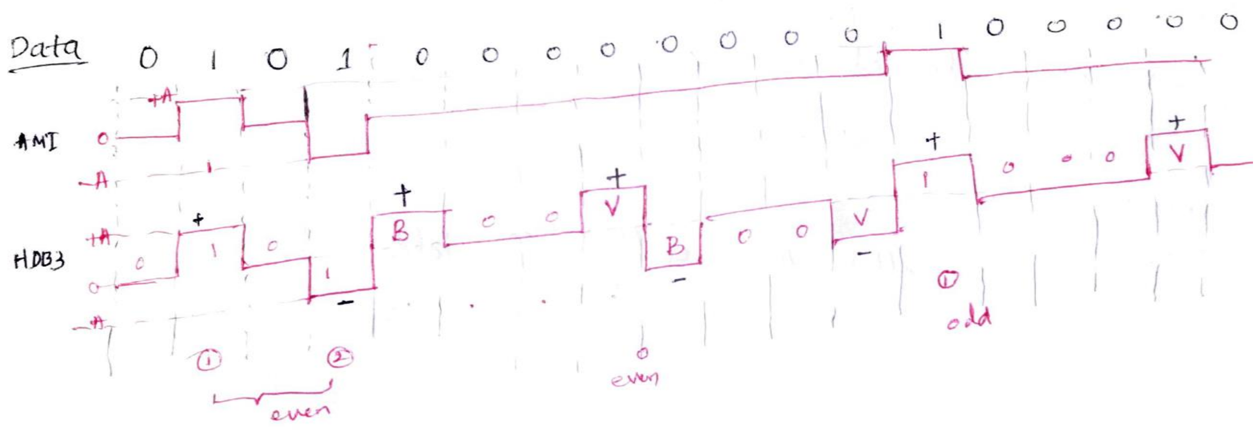
$$\hookrightarrow = -\cos 2\pi f_c t$$

(1)

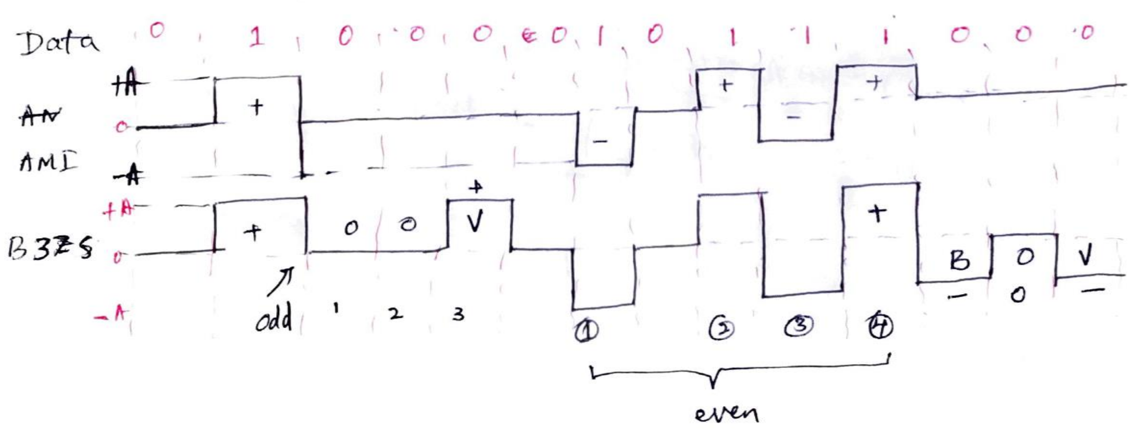
Ex: 1 HDB3 coding



Ex2:

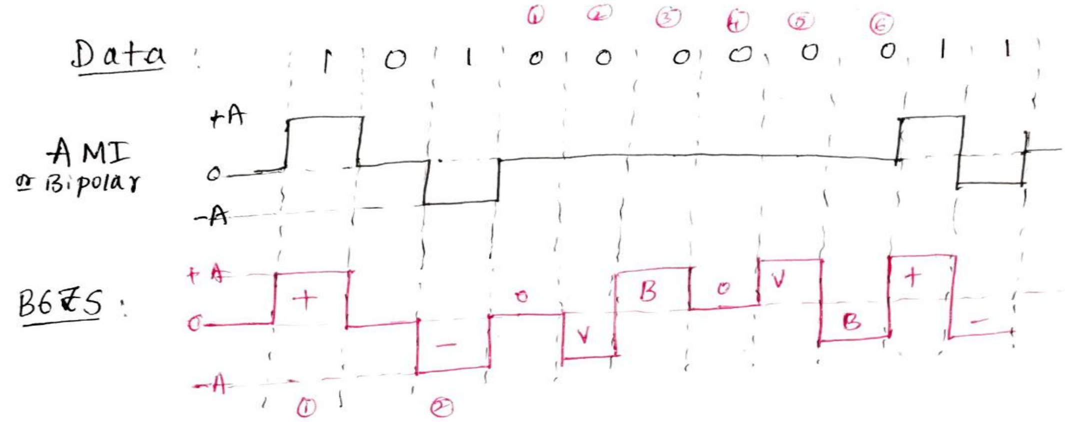


Ex3: B3ZS coding



Ex 4:
B6ZS - coding -

(2)



Problems on pre-envelope and complex envelope :

(1) Determine the pre-envelope and complex envelope of the RF pulse defined by,

$$x(t) = A \ \text{rect}\left(\frac{t}{T}\right) \cdot \cos(2\pi f_c t)$$

Soln :

given $x(t) = A \cdot \underbrace{\text{rect}\left(\frac{t}{T}\right)}_{\substack{\text{low freq} \\ m(t)}} \cdot \underbrace{\cos(2\pi f_c t)}_{\substack{\text{high freq.} \\ c(t)}}$ — (1)

Pre-envelope can be calculated using eq 2.

$$x_+(t) = x(t) + j\hat{x}(t)$$

$$x_+(t) = A \cdot \text{rect}\left(\frac{t}{T}\right)\left[\cos 2\pi f_c t + j\sin(2\pi f_c t)\right] \quad \left(\because \ m(t)\cos(2\pi f_c t) \xrightarrow{H.T} m(t)\sin 2\pi f_c t\right)$$

$$\therefore \ \boxed{x_+(t) = A \ \text{rect}\left(\frac{t}{T}\right) \cdot e^{j2\pi f_c t}} \rightarrow (2) - \text{pre-envelope of the pulse.}$$

Now, to determine complex envelope, we use eq 2,

$x_+(t) = \tilde{x}(t) e^{j2\pi f_c t}$ — (3)

$$\Rightarrow \tilde{x}(t) = x_+(t) e^{-j2\pi f_c t}$$

$$= A \cdot \text{rect}\left(\frac{t}{T}\right) \cancel{e^{j2\pi f_c t}} \cdot \cancel{e^{-j2\pi f_c t}}$$

$$\boxed{\tilde{x}(t) = A \ \text{rect}\left(\frac{t}{T}\right)} \text{ — (3) complex envelope of the pulse.}$$

(3)

(2) Determine pre-envelope and complex envelope of the signal given by,

$s(t) = e^{-at}[\cos(\omega_c+\Delta\omega)t]\, u(t)$

Soln:

$$s(t) = e^{-at}[\cos(\omega_c+\Delta\omega)t]\, u(t) \quad -(1)$$

Eqn (1) can be written as,

$$s(t) = \underbrace{e^{-at}\, u(t)}_{m(t)}\ [\cos \underbrace{(\omega_c+\Delta\omega)t}_{c(t)}]$$

($m(t)\cos 2\pi f_c t \xrightarrow{H.T} m(t)\sin 2\pi f_c t$

& $m(t)\sin 2\pi f_c t \rightarrow -m(t)\cos 2\pi f_c t$)

Pre-envelope -

$$s_+(t) = s(t) + j\hat{s}(t)$$

$$= e^{-at}u(t)[\cos(\omega_c+\Delta\omega)t] + j e^{-at}u(t)[\sin(\omega_c+\Delta\omega)t]$$

$$= e^{-at}u(t)[\cos(\omega_c+\Delta\omega)t + j\sin(\omega_c+\Delta\omega)t]$$

$$= e^{-at}u(t)\, e^{j(\omega_c+\Delta\omega)t}$$

$$\boxed{s_+(t) = e^{-at}\, e^{j(\omega_c+\Delta\omega)t}\, u(t).}$$ — Pre-envelope pulse

complex envelope:

w.k.t.

$$s_+(t) = \tilde{s}(t)\, e^{+j\omega_c t}$$

$$\Rightarrow \tilde{s}(t) = s_+(t)\, e^{-j\omega_c t}$$

$$= e^{-at}\, e^{+j\omega_c t + j\Delta\omega t}\, u(t)\, e^{-j\omega_c t}$$

$$= e^{-at}\, \cancel{e^{j\omega_c t}}\, e^{j\Delta\omega t}\, \cancel{e^{-j\omega_c t}}\, u(t)$$

$$\boxed{\tilde{s}(t) = e^{-at}\, e^{j\Delta\omega t}\, u(t)}$$ ← complex envelope.